

The Development of Baryon Asymmetry in the Early Universe

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ABSTRACT

The development of an excess of baryons over antibaryons due to CP and baryon number violating reactions during the very early stages of the big bang is calculated in simple models using the Boltzmann equation.

There are observational and theoretical indications that the local preponderance of baryons over antibaryons extends throughout the universe (at least since the time when the temperature $T \sim 100$ MeV) with an average ratio of baryon to photon densities [1] $n_B/n_\gamma \equiv Y_B \approx 10^{-9}$. If baryon number (B) were absolutely conserved in all processes, this small baryon excess must have been present since the beginning of the universe. However, many grand unified gauge models [2] require superheavy particles (typically with masses $m_X \sim 10^{15}$ GeV $\equiv 1$ HeV) which mediate baryon- and lepton-number (L) violating interactions. Any direct evidence for these must presumably come from an observation of proton decay. In the standard hot big bang model [1], the temperature T (of light particle species) in the early universe fell with time t according to (taking units such that $\hbar = c = k = 1$) $T \sim \sqrt{m_p/2t}$ where $m_p = (45/8\pi^3)^{1/2} m_\phi/\sqrt{\xi(T)} \approx 5 \times 10^3/\sqrt{\xi}$ MeV, and $m_\phi = G^{-1/2} \approx 10^{19}$ GeV is the Planck mass, while ξ gives the effective number of particle species in equilibrium ($\xi = \frac{1}{2}(\frac{7}{16})$ for each ultrarelativistic boson (nondegenerate fermion) spin state). At temperatures $T \gtrsim m_X$, B-violating interactions should have been important, and they should probably have destroyed or at least much diminished any initial baryon excess. (This occurs even when, for example, B-L is absolutely conserved, since then an initial baryon excess would presumably be accompanied by a lepton excess, so as to maintain the accurate charge neutrality of the universe.) It is interesting (and in some models necessary) to postulate that B-violating interactions in the very early universe could give rise to a calculable baryon excess even from an initially symmetrical state. For this to be possible, the rates for reactions producing baryons and antibaryons must differ, and hence the interactions responsible must violate CP invariance. We describe here a simple but general method for calculating B generation in any specific model. We clarify and extend previous estimates [3].

Let $M(i \rightarrow j)$ be the amplitude for transitions from the state i to j , and let \bar{i} be the CP conjugate of i (particles \rightarrow antiparticles and spins reversed). Then CPT invariance demands $M(i \rightarrow j) = M(\bar{j} \rightarrow \bar{i})$, while CP invariance would require $M(i \rightarrow j) = M(\bar{i} \rightarrow \bar{j}) = M(j \rightarrow i)$. Unitarity [4] (transitions to and from i must occur with total probability 1) demands $\sum_j |M(i \rightarrow j)|^2 = \sum_j |M(j \rightarrow i)|^2$; combining this with the constraint of CPT invariance yields (the sum over j includes all states and their antistates)

$$\sum_j |M(i \rightarrow j)|^2 = \sum_j |M(\bar{i} \rightarrow j)|^2 = \sum_j |M(j \rightarrow \bar{i})|^2 = \sum_j |M(j \rightarrow i)|^2. \quad (1)$$

In thermal equilibrium (and in the absence of chemical potentials representing nonzero conserved quantum numbers) all states j of a system with a given energy are equally populated. Then the last equality in (1) shows that transitions from these states (interactions) must produce i and \bar{i} in equal numbers; thus no excess of particles over antiparticles may develop in a system in thermal equilibrium, even if CP is violated. In addition, the first equality in (1) shows that the total cross-sections for destroying particles and antiparticles must be equal. Since in thermal equilibrium no excess of i over \bar{i} may develop, this implies that any initial excess must be destroyed.

The phase space distribution $f_i(p)$ (number per unit cell $d^3p d^3x$ [5]) for a species i develops with time (on average) according to a Boltzmann transport equation. A closed system with no external influences obeys Boltzmann's H theorem (which holds regardless of T (i.e., CP) invariance [6]), so that from any initial state the $f_i(p)$ evolve (on average) to their equilibrium forms for which $f_{\bar{i}}(p) = f_i(p)$, and no baryon excess may survive.

However, in an expanding universe, extra terms must be added to the Boltzmann equations, and if some participating particles are massive [7], a baryon excess may be generated; the relaxation time necessary to destroy the excess often increases faster than the age of the universe [8].

Equation (1) requires that the total rates for processes with particle and antiparticle initial states be equal. CP violation allows the rates for specific conjugate reactions to differ; unitarity nevertheless requires ($T = i(1-S)$, $SS^\dagger = S^\dagger S = 1$) [9]:

$$\begin{aligned} |M(i \rightarrow j)|^2 - |M(\bar{i} \rightarrow \bar{j})|^2 &= |T_{ij}|^2 - |T_{ji}|^2 \\ &= 2 \operatorname{Im}[(\sum_n TT^\dagger)_{ij} T_{ji}^*] - |(\sum_n TT^\dagger)_{ij}|^2. \end{aligned} \quad (2)$$

Hence the fractional difference between conjugate rates must be at least $O(\alpha)$ where α is some coupling constant [10]. Moreover, the loop diagrams giving CP violation must allow physical intermediate states n . (These loop corrections must usually also be B-violating to give a difference in rates when summed over all final states $\binom{-}{j}$ with a given $(-)B$ [11].)

Let $\binom{-}{b}$ be an "(anti)baryon" with $B = \begin{matrix} + \\ - \end{matrix} \frac{1}{2}$. For simplicity we assume here that all particles (including photons) obey Maxwell-Boltzmann statistics and have only one spin state. In our first (very simple) model, we consider CP, B violating $2 \leftrightarrow 2$ reactions involving $\binom{-}{b}$ and a heavy neutral particle ϕ ; we take their rates to be (this parametrization ensures unitarity and CPT invariance)

$$\begin{aligned} |M(bb \rightarrow \bar{b}\bar{b})|^2 &= (1 + \zeta) |M_o|^2/2 \\ |M(bb \rightarrow \phi\phi)|^2 &= |M(\phi\phi \rightarrow \bar{b}\bar{b})|^2 = (1 - \zeta) |M_o|^2/2 \\ |M(\bar{b}\bar{b} \rightarrow bb)|^2 &= (1 + \bar{\zeta}) |M_o|^2/2 \end{aligned}$$

$$|M(\bar{b}\bar{b} \rightarrow \phi\phi)|^2 = |M(\phi\phi \rightarrow bb)|^2 = (1-\bar{\zeta}) |M_0|^2/2 \quad (3)$$

where $\zeta-\bar{\zeta} = O(\alpha)$ measures the magnitude of CP violation. The number of a species i per unit volume $n_i \equiv \int d^3p/(2\pi)^3 f_i(p)$ decreases with time even without collisions in an expanding universe according to (R is the Robertson-Walker scale factor; dots denote time derivatives)

$$\frac{dn_i}{dt} = n_i \frac{d(1/V)}{dt} = -\frac{3\dot{R}}{R} n_i = \frac{3\dot{T}}{T} n_i = -\frac{3T^2}{M_P^2} n_i. \quad (4)$$

The n_i are also changed by collisions; the (average) time development of the ϕ and baryon number ($n_B \equiv n_b - n_{\bar{b}}$) densities is given by the Boltzmann equations ($Y_i \equiv n_i/n_\gamma$ where γ is a massless particle; $|M_0|^2 = O(\alpha^2)$)

$$\begin{aligned} n_{\gamma} \dot{Y}_\phi &= \dot{n}_\phi + \frac{3\dot{R}}{R} n_\phi = 2\Lambda_{12}^{34} \left[f_b(p_1) f_b(p_2) |M(bb \rightarrow \phi\phi)|^2 \right. \\ &\quad + \frac{f_b(p_1) f_b(p_2)}{b} |M(\bar{b}\bar{b} \rightarrow \phi\phi)|^2 - f_\phi(p_1) f_\phi(p_2) (|M(\phi\phi \rightarrow bb)|^2 \\ &\quad \left. + |M(\phi\phi \rightarrow \bar{b}\bar{b})|^2) \right] \end{aligned} \quad (5a)$$

$$\begin{aligned} n_{\gamma} \dot{Y}_B &= \Lambda_{12}^{34} \left[-f_b(p_1) f_b(p_2) (2|M(bb \rightarrow \bar{b}\bar{b})|^2 + |M(bb \rightarrow \phi\phi)|^2) \right. \\ &\quad + \frac{f_b(p_1) f_b(p_2)}{b} (2|M(\bar{b}\bar{b} \rightarrow bb)|^2 + |M(\bar{b}\bar{b} \rightarrow \phi\phi)|^2) \\ &\quad \left. + f_\phi(p_1) f_\phi(p_2) (|M(\phi\phi \rightarrow bb)|^2 - |M(\phi\phi \rightarrow \bar{b}\bar{b})|^2) \right], \end{aligned} \quad (5b)$$

where the operator Λ represents suitable integration over initial and final state momenta. We assume that the b undergo baryon-conserving collisions with a frequency much higher than the $O(\alpha^3)$ rate on which n_B changes (as is presumably the case in realistic models). They are therefore always in kinetic

equilibrium with the rest of the universe, and hence Maxwell-Boltzmann distributed in phase space:

$$f_{(-)}(p) \approx \exp[-(E_{(-)}\mu)/T]$$

$$Y_B \equiv (n_b - n_{\bar{b}})/n_Y \approx 2 \sinh(\mu/T) . \quad (6)$$

μ is a baryon number chemical potential, which is changed only by B-violating processes, and would vanish if chemical equilibrium prevailed. Assuming $Y_B \ll 1$, one may use momentum conservation in (5) to write $f_{(-)}(p_1) f_{(-)}(p_2) \approx \exp[-(E_3+E_4)/T](1+Y_B) \approx f_{\phi}^{\text{eq}}(p_3) f_{\phi}^{\text{eq}}(p_4) (1+Y_B)$, where $f_{\phi}^{\text{eq}}(p) = \exp(-E/T)$ is the equilibrium distribution of ϕ at temperature T : The equilibrium ϕ number density $n_{\phi}^{\text{eq}} = T^3/(2\pi^2)(m_{\phi}/T)^2 K_2(m_{\phi}/T)$, where K_2 is a modified Bessel function [12] (as $m_{\phi} \rightarrow 0$, $n_{\phi}^{\text{eq}} \rightarrow T^3/\pi^2$; as $T \rightarrow 0$, $n_{\phi}^{\text{eq}} \rightarrow (m_{\phi}T/2\pi)^{3/2} \exp(-m_{\phi}/T)$). Then substituting the parametrization (3) and performing phase space integrations, (5) becomes

$$\dot{Y}_{\phi} \approx n_Y \langle \sigma_{\phi} v \rangle \left\{ 2 \left[1 - \left(\frac{\zeta + \bar{\zeta}}{2} \right) \right] \left[(Y_{\phi}^{\text{eq}})^2 - Y_{\phi}^2 \right] - (\zeta - \bar{\zeta}) (Y_{\phi}^{\text{eq}})^2 Y_B \right\} , \quad (7a)$$

$$\dot{Y}_B \approx n_Y \langle \sigma_{\phi} v \rangle \left\{ \left(\frac{\zeta - \bar{\zeta}}{2} \right) \left[Y_{\phi}^2 - (Y_{\phi}^{\text{eq}})^2 \right] - \left[3 + \left(\frac{\zeta + \bar{\zeta}}{2} \right) \right] (Y_{\phi}^{\text{eq}})^2 Y_B \right\} , \quad (7b)$$

where $\langle \sigma_{\phi} v \rangle$ is the cross-section corresponding to $|M_0|^2$ averaged over a flux of incoming particles in equilibrium energy distributions. Equation (7b) exhibits the necessity of deviation from equilibrium for B generation, and the destruction of Y_B in equilibrium.

We now turn to a slightly more realistic but more complicated model in

which massive particles $\overset{(-)}{X}$ decay to $\overset{(-)}{b}$ with rates ($\gamma_X = O(\alpha)$)

$$\begin{aligned} |M(X \rightarrow bb)|^2 &= |M(\bar{b}\bar{b} \rightarrow \bar{X})|^2 = (1+\eta)\gamma_X/2 \\ |M(X \rightarrow \bar{b}\bar{b})|^2 &= |M(bb \rightarrow \bar{X})|^2 = (1-\eta)\gamma_X/2 \\ |M(\bar{X} \rightarrow bb)|^2 &= |M(\bar{b}\bar{b} \rightarrow X)|^2 = (1-\bar{\eta})\gamma_X/2 \\ |M(\bar{X} \rightarrow \bar{b}\bar{b})|^2 &= |M(bb \rightarrow X)|^2 = (1+\bar{\eta})\gamma_X/2 \end{aligned} \quad (8)$$

Note that if $\overset{(-)}{X}$ decays preferentially produce b , then CPT invariance implies that \bar{b} are preferentially destroyed in inverse processes; thus $\overset{(-)}{X}$ decays and inverse decays (DID) alone would generate a net B even if all particles were in thermal equilibrium, in contravention of the theorem (1) [13]. However, the CP violation parameter ($\eta-\bar{\eta}$) is $O(\alpha)$, and hence changes in n_B from DID are of the same order as $2 \rightarrow 2$ scattering processes, such as $bb \rightarrow \bar{b}\bar{b}$. It will turn out that s -channel exchange of nearly on-shell $\overset{(-)}{X}$ in $\overset{(-)}{b} \overset{(-)}{b} \leftrightarrow bb$ cancels the DID contribution to \dot{Y}_B so as to recover $\dot{Y}_B = 0$ in thermal equilibrium. In direct analogy with eqs. (5) and (7), and using the assumption (6), the equation for the evolution of the $\overset{(-)}{X}$ number density

$n_{\overset{(-)}{X}} \equiv Y_{\overset{(-)}{X}} n_Y$ becomes

$$\dot{Y}_{\overset{(-)}{X}} = -\langle \Gamma_X \rangle \{ (Y_X - Y_X^{eq}) - \bar{\eta} Y_B Y_X^{eq} \} ; \quad (9)$$

the corresponding equation for $\dot{Y}_{\bar{X}}$ is obtained by charge conjugation ($Y_X \leftrightarrow Y_{\bar{X}}, Y_B \rightarrow -Y_B, \eta \leftrightarrow \bar{\eta}$). The $\langle \Gamma_X \rangle$ in (9) is the total $\overset{(-)}{X}$ decay width multiplied by the time dilation factor m_X/E_X and averaged over the equilibrium X energy distribution [14]. The baryon concentration evolves according to

$$\begin{aligned}
\dot{Y}_B &= \langle \Gamma_X \rangle \{ \eta Y_X - \bar{\eta} Y_{\bar{X}} + (\eta - \bar{\eta}) Y_X^{\text{eq}} - 2Y_B Y_X^{\text{eq}} \} \\
&- \frac{2}{n_Y} \Lambda_{12}^{34} \{ f_X^{\text{eq}}(p_1 + p_2) (|M'(bb \rightarrow \bar{b}\bar{b})|^2 \\
&- |M'(\bar{b}\bar{b} \rightarrow bb)|^2) \\
&- 2Y_B \Lambda_{12}^{34} \{ f_X^{\text{eq}}(p_1 + p_2) (|M'(bb \rightarrow \bar{b}\bar{b})|^2 + |M'(\bar{b}\bar{b} \rightarrow bb)|^2) \} \quad (10)
\end{aligned}$$

where the first term is from DID (and does not separately vanish when $Y_X^{(-)} = Y_X^{\text{eq}}$), while the second two terms arise from $2 \rightarrow 2$ scatterings. The DID term accounts for sequential inverse decay and decay processes involving real X : these are therefore subtracted from the true $2 \rightarrow 2$ scattering terms by writing $|M'(i \rightarrow j)|^2 = |M(i \rightarrow j)|^2 - |M_{\text{RIX}}(i \rightarrow j)|^2$, where $M_{\text{RIX}}(i \rightarrow j)$ is the amplitude for $i \rightarrow j$ due to on-shell s channel X exchange. In the narrow X width approximation, $|M_{\text{RIX}}(i \rightarrow j)|^2 \sim |M(i \rightarrow X^{(-)})|^2 |M(X^{(-)} \rightarrow j)|^2 / \Gamma_X$; the presence of the Γ_X denominator renders it $O(\alpha)$. According to the theorem (1), the CP violating difference of total rates $|M(bb \rightarrow \bar{b}\bar{b})|^2 - |M(\bar{b}\bar{b} \rightarrow bb)|^2 = O(\alpha^3)$. Hence $|M'(bb \rightarrow \bar{b}\bar{b})|^2 - |M'(\bar{b}\bar{b} \rightarrow bb)|^2 = |M_{\text{RIX}}(\bar{b}\bar{b} \rightarrow bb)|^2 - |M_{\text{RIX}}(bb \rightarrow \bar{b}\bar{b})|^2 + O(\alpha^3) = O(\alpha^2)$, and the second term in eq. (10) becomes $-2\langle \Gamma_X \rangle (\eta - \bar{\eta}) Y_X^{\text{eq}}$, thereby elegantly cancelling the first term in thermal equilibrium. Finally, therefore,

$$\begin{aligned}
\dot{Y}_B &= \langle \Gamma_X \rangle \{ (\eta - \bar{\eta}) [(Y_X + Y_{\bar{X}})/2 - Y_X^{\text{eq}}] \\
&+ (\eta + \bar{\eta}) (Y_X - Y_{\bar{X}})/2 \} \\
&- 2Y_B \{ \langle \Gamma_X \rangle Y_X^{\text{eq}} + n_Y \langle v\sigma' (bb \rightarrow \bar{b}\bar{b}) + v\sigma' (\bar{b}\bar{b} \rightarrow bb) \rangle \} \quad (11)
\end{aligned}$$

The differential equations (10) and (11) must now be solved with the initial condition $Y_X(t=0) = Y_X^{\text{eq}}(0)$, and possibly an initial baryon density

Y_B . Figure 1 shows the solutions with guesses for parameters based on the SU(5) model [2] ($m_X = 10^{15}$ GeV and 10^{14} GeV; $\alpha \approx 1/40$ (vector decays), or 10^{-3} (scalar decays)). If all X initially in thermal equilibrium decayed with no back reactions, the Y_B generated would be simply $\eta - \bar{\eta}$. For small α or large m_X/m_P this upper limit is approached. (At small $x \equiv m_X/T$, series solution of equations (10) and (11) gives $(Y_X + Y_{\bar{X}})/2 \approx 1 - ax^5/20$; $(Y_X - Y_{\bar{X}})/2 \approx (\eta - \bar{\eta})a^2 x^8/160$; $Y_B \approx (\eta - \bar{\eta})ax^5/20$, where $a = m_P \Gamma_X/m_X^2$.) For $T \ll m_X$, baryon number is destroyed by $2 \rightarrow 2$ reactions with $\sigma \sim \alpha^2 T^2/m_X^4$ roughly like $Y_B(T) \sim \exp[\alpha^2 m_P T^3/m_X^4]$ [15], so that $Y_B \rightarrow \text{constant}$ as $T \rightarrow 0$, but if m_X is small, the final Y_B is much diminished from its value at higher T . The Y_B generated is always roughly linearly proportional to $\eta - \bar{\eta}$, but is a sensitive function of m_X/m_P and α ; for realistic values of these parameters, a numerical solution is probably essential.

According to equation (11), any baryon excess existing at the Planck time $t_P = 1/m_P$ should be diminished by inverse decays at $T \gg m_X$ so that $Y_B(t)/Y_B(t_P) \sim \exp[-\alpha m_X m_P/T^2]$; any initial Y_B should be reduced by a factor $\sim \exp[-m_P/m_X]$ before CP violating processes can generate Y_B at $T \lesssim m_X$. B-violating $2 \rightarrow 2$ scatterings at temperatures $m_P > T > m_X$ should reduce an initial Y_B by a factor $\sim \exp[-m_P \int_{m_P}^{m_X} \langle v\sigma \rangle dT]$. One might expect that $\langle v\sigma \rangle \sim \alpha^2/m_X^2$ at high energies due to t-channel vector X exchange; however, the effective $\langle v\sigma \rangle$ presumably relevant for the Boltzmann equation is rather $\langle v\sigma_{\text{eff}} \rangle \sim \alpha^2/\lambda_D^2$ where the Debye screening length $\lambda_D \sim [\sqrt{32\alpha} T]^{-1}$. In this approximation $2 \rightarrow 2$ and higher multiplicity collisions are probably no more effective at destroying an initial Y_B than are inverse decays.

We conclude therefore that B-violating reactions in the very early universe might well destroy any initial baryon number existing around the Planck time ($1/m_P$), requiring subsequent B and CP-violating interactions

to generate the observed baryon asymmetry. The methods described here [16] allow a calculation of the resulting baryon excess in any specific model; the simple examples considered suggest that the observed Y_B should place stringent constraints on parameters of the model.

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4. V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, Relativistic Quantum Theory (Pergamon, Oxford, 1971), p. 239. In the present context the importance of unitarity was emphasized by Toussaint, et al., ref. 3.
5. We assume homogeneity and isotropy, so that $f(\vec{p}, x) = f(\vec{p}) = f(p)$.
6. See, e.g., A. Aharony in Modern Developments in Thermodynamics (Wiley, New York, 1973), pp. 95-114.
7. It is not necessary that these participate directly in B violating reactions.
8. In the simple models discussed below, this phenomenon occurs if the universe is homogeneous and always cools faster than $T \sim m_p / (tm_p)^{1/5}$; in practice the quark excess will always be contained in baryons where their probability for collisions remains constant rather than falling as in a homogeneous expanding universe.

9. This constraint applies only if no initial or final particles may mix with their antiparticles (as in the K^0 system). CP-violating mixing requires a difference $M(i \rightarrow j) - M(\bar{i} \rightarrow \bar{j}) \neq 0$ in amplitudes rather than rates.
10. Regardless of perturbation theory, CP violation is asymptotically suppressed by powers of $\log(s)$, where \sqrt{s} is the invariant mass of the initial state.
11. D. V. Nanopoulos and S. Weinberg, "Mechanisms for Cosmological Baryon Production," Harvard University Preprint 79/4023.
12. I. S. Gradestyn and I. M. Ryzhik, Table of Integrals Series and Products (Academic Press, New York, 1965), p. 951.
13. This rather relevant point has also been noticed by A. D. Dolgov and Ya. B. Zeldovich (ref. 3), but was apparently neglected elsewhere.
14. Strictly, m_X/E_X should be averaged separately for the various terms of (9); if X is in kinetic equilibrium, however, the averages are equal. Note that we have implicitly assumed all produced and decaying X to be exactly on their mass shells. However, particularly at high T, the mean X collision time $\ll 1/\Gamma_X$, so that the X resonance is collision broadened, and produced or decaying X may be far off shell. The m_X/E_X factor for inverse decays essentially arises from the fact that the incoming particles must subtend a sufficiently small angle to have invariant mass m_X ; if produced X are far off shell, the m_X/E_X in DID should disappear.
15. If $T \sim m_p/(tm_p)^{1/5}$ as in footnote 8, then $Y_B \sim \exp(-1/T)$; the universe expands sufficiently slowly for Y_B to relax to zero.
16. Many details are given in E. W. Kolb, and S. Wolfram, "Baryon Number Generation in the Early Universe," Caltech report OAP 547.

17. Results depend only on m_X through the dimensionless combination m_X/m_P ; here we take $\xi = 100$ in the definition of m_P . Note that inhomogeneities in the early universe may be manifest in different expansion rates and hence different effective ξ for different regions. The final Y_B produced could vary considerably between the regions.

Figure Caption

The development of baryon number density (solid curves) as a function of inverse temperature in the model of eq. (11) for various choices of parameters (unless otherwise indicated, $\alpha = 1/40$ and $m_X = 1 \text{ PeV} \equiv 10^{15} \text{ GeV}$ [17]). The dashed and dotted curves give $(Y_X + \bar{Y}_X)/2$ and $(Y_X - \bar{Y}_X)/2$, respectively. In all cases we have taken the CP violation parameter, $\eta - \bar{\eta} = 10^{-6}$.

