states.

There is thus no polarization state that can go through a longitudinal and then a transverse shift amplifier (or the contrary). In other words, as these two shifts have no common eigenfunctions, their eigenvalues cannot be simultaneously displayed.

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Foundations for a Theory of Gravitation Theories*

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A foundation is laid for future analyses of gravitation theories. This foundation is applicable to any theory formulated in terms of geometric objects defined on a 4-dimensional spacetime manifold. The foundation consists of (i) a glossary of fundamental concepts, (ii) a theorem that delineates the overlap between Lagrangian-based theories and metric theories; (iii) a conjecture (due to Schiff) that the weak equivalence principle implies the Einstein equivalence principle; and (iv) a plausibility argument supporting this conjecture for the special case of relativistic, Lagrangian-based theories.

I. INTRODUCTION

Several years ago our group initiated[1] a project of constructing theoretical foundations for experimental tests of gravitation theories. The results of that project to date (largely due to Will and Ni) and the results of a similar project being carried out by the group of Nordtvedt at Montana State University are summarized in several recent review articles.[2-4] Those results have focused almost entirely on "metric theories of gravity" (relativistic theories that embody the Einstein equivalence principle; see Sec. III below).

By January 1972, metric theories were suffi-

ciently well understood that we began to broaden our horizons to include nonmetric theories. The most difficult aspect of this venture has been communication. The basic concepts used in discussing nonmetric theories in the past have been defined so vaguely that discussions and "cross-theory analyses" have been rather difficult. To remedy this situation we have been forced, during these last eleven months, to make more precise a number of old concepts and to introduce many new ones. By trial and error, we have gradually built up a glossary of concepts that looks promising as a foundation for analyzing nonmetric theories.

Undoubtedly we shall want to change some of our concepts, and make others more precise, as we proceed further. But by now our glossary is sufficiently stabilized, and we have derived enough interesting results using it, that we feel compelled to start publishing.

This paper presents the current version of our glossary (Secs. II–IV), and uses it to outline some key ideas and results about gravitation theories, both nonmetric and metric (Secs. V and VI). Subsequent papers will explore some of those ideas and results in greater depth.

Central to our current viewpoint on gravitation theories is the following empirical fact. Only two ways have ever been found to mesh a set of gravitational laws with all the classical, special relativistic laws of physics. One way is the route of the Einstein equivalence principle (EEP) – (i) Describe gravity by one or more gravitational fields, including a metric tensor $g_{\alpha\beta}$; and (ii) insist that in the local Lorentz frames of $g_{\alpha\beta}$ all the nongravitational laws take on their standard special relativistic forms. The second way of meshing is the route of the Lagrangian – (i) Take a special relativistic Lagrangian for particles and nongravitational fields, and (ii) insert gravitational fields into that Lagrangian in a manner that retains general covariance. The equivalence-principle route always leads to a metric theory. (Example: general relativity.) The Lagrangian route always leads to a "Lagrangian-based theory." [Example: Belinfante-Swihart theory (Table IV, later in this paper).] Thus, in the future we expect most of our attention to focus on metric theories and on Lagrangian-based theories; and in the nonmetric case we might be able to confine attention to theories with Lagrangians.

Since metric theories are so well understood,[4] it would be wonderful if one could prove that all nonmetric, Lagrangian-based theories are defective in some sense. A conjecture due to Schiff[5] points to a possible defect. Schiff's conjecture says[5] that *any complete and self-consistent theory that obeys the weak equivalence principle (WEP) must also, unavoidably, obey the Einstein equivalence principle (EEP).* (See Sec. III for precise definitions.) Since any relativistic, Lagrangian-based theory that obeys EEP is a metric theory, this conjecture suggests that nonmetric, relativistic, Lagrangian-based theories should always violate WEP.

The experiments of Eötvös *et al.*[6] and Dicke *et al.*,[7] with modifications by Braginsky *et al.*[8] (ED experiments), are high-precision tools for testing WEP. Hence, the Schiff conjecture suggests that, if one has a nonmetric Lagrangian-based theory, one should test whether it violates the ED experiments. (Such tests for the Belinfante-Swihart and Naida-Capella theories reveal violations of ED and WEP.[9])

In this paper, after presenting our glossary of concepts (Secs. II–IV), we shall (i) derive a criterion for determining whether a Lagrangian-based theory is a metric theory ("principle of universal coupling," Sec. V), and (ii) discuss and make plausible Schiff's conjecture (Sec. VI).

II. CONCEPTS RELEVANT TO SPACETIME THEORIES

This section, together with Secs. III and IV, presents our glossary of concepts. To understand these concepts fully, the reader should be familiar with the foundations of differential geometry as laid out, for example, by Trautman.[10] He should also be familiar with Chap. 4 of Anderson's textbook[11] (cited henceforth as JLA), from which we have borrowed many concepts. However, he should notice that we have modified slightly some of JLA's concepts, and we have reexpressed some of them in the more precise notation and terminology of Trautman[10] and of Misner, Thorne, and Wheeler (MTW).[12]

The concepts introduced in this section apply to any "spacetime theory" (see below for definition). In Secs. III and IV we shall specialize to "gravitation theories," which are a particular type of spacetime theory. To make our concepts clear, we shall illustrate them using four particular gravitation theories: the Newton-Cartan theory (Table I), general relativity (Table II), Ni's theory (Table III),[13] and the Belinfante-Swihart theory (Table IV).[14,15] Of these theories, general relativity and Ni's theory are metric; the Newton-Cartan and Belinfante-Swihart theories are nonmetric.

Mathematical representations of a theory. Two different mathematical formalisms will be called "different representations of the same theory" if they produce identical predictions for the outcome of every experiment or observation. Here by "outcome of an experiment or observation" we mean

TABLE I. Newton-Cartan theory.

1. Reference for this version of the theory:

   Chapter 12, and especially Box 12.4 of MTW[12]

2. Gravitational fields.

   a. Symmetric covariant derivative (affine connection) ..... $\underline{\nabla}$
   b. Spatial metric ..... $\underline{\gamma}$
   c. Universal time ..... t

3. Gravitational field equations:

   a. $\underline{\nabla}\, \underline{dt} = 0$.
   b. $\mathcal{R}(\underline{u}, \underline{n})\underline{w} = 0$,

      where $\mathcal{R}$ is the curvature operator formed from $\underline{\nabla}$; $\underline{u}$ and $\underline{n}$ are arbitrary vectors; $\underline{w}$ is any spatial vector ($\langle \underline{dt}, \underline{w} \rangle = 0$).
   c. $\mathcal{R}(\underline{v}, \underline{w}) = 0$
      for every pair of spatial vectors, $\underline{v}, \underline{w}$. [*Note:* a, b, c guarantee the existence of the metric, $\underline{\gamma}$ or "$\cdot$", defined on spatial vectors only, such that

      $$\underline{\nabla}_{\underline{u}}(\underline{w} \cdot \underline{v}) = (\underline{\nabla}_{\underline{u}} \underline{w}) \cdot \underline{v} + \underline{w} \cdot (\underline{\nabla}_{\underline{u}} \underline{v})$$

      for any $\underline{u}$ and for any spatial $\underline{w}, \underline{v}$.]
   d. $v \cdot [\mathcal{J}(\underline{u}, \underline{n})\underline{w}] = w \cdot [\mathcal{J}(\underline{u}, \underline{n})\underline{v}]$
      for all spatial $\underline{v}, \underline{w}$ and for any $\underline{u}, \underline{n}$, where

      $$\mathcal{J}(\underline{u}, \underline{n})\underline{p} \equiv \tfrac{1}{2}[\mathcal{R}(\underline{p}, \underline{n})\underline{u} + \mathcal{R}(\underline{p}, \underline{u})\underline{n}] .$$

   e. $\underline{\text{Ricci}} = 4\pi\rho\, \underline{dt} \otimes \underline{dt}$,
      where $\underline{\text{Ricci}}$ is the Ricci tensor formed from $\underline{\nabla}$, and ρ is mass density.

4. Influence of gravity on matter:

   a. Test particles move along geodesics of $\underline{\nabla}$, with t an affine parameter.
   b. Each test particle carries a local *inertial frame* with orthonormal, parallel-transported spatial basis vectors ($\underline{e}_j \cdot \underline{e}_k = \delta_{jk}$, $\underline{\nabla}_{\underline{u}} \underline{e}_j = 0$) and with $\underline{e}_{\hat{0}} = d/dt$ = (tangent to geodesic world line).
   c. All the nongravitational laws of physics take on their standard, Newtonian forms in every local inertial frame.

the raw numerical data, before interpretation in terms of theory. Any theory can be given a variety of different mathematical representations. [Example – The Dicke-Brans-Jordan theory has two "standard representations: (i) the original representation,[16,17] in which test particles move on geodesics but the field equations differ significantly from those of Einstein; and (ii) the conformally transformed representation,[18] in which the scalar field produces deviations from geodesic motion but the field equations are nearly the same as Einstein's.] A theory can be regarded as the

TABLE II. General relativity theory.

1. Reference: Standard textbooks, e.g., MTW.[12]

2. Gravitational field:

   The metric of spacetime ..... $\underline{g}$

3. Gravitational field equations:

   $$\underline{G} = 8\pi \underline{T} ,$$

   where $\underline{G}$ is the Einstein tensor formed from $\underline{g}$, and $\underline{T}$ is the stress-energy tensor.

4. Influence of gravity on matter:

   a. Test particles move along geodesics of $\underline{g}$, with proper time τ an affine parameter.
   b. Each test particle carries a local inertial ("local Lorentz") frame with parallel-transported, orthonormal basis vectors $\underline{e}_{\hat{\alpha}}$, and with $\underline{e}_{\hat{0}} = d/d\tau$ = (tangent to geodesic world line).
   c. All the nongravitational laws of physics take on their standard, special-relativistic forms in every local inertial frame (aside from delicate points associated with "curvature coupling"; see Chap. 16 of MTW[12]).

TABLE III. Ni's "New Theory."

1. Reference: Ni[13]

2. Gravitational fields:

   a. Background metric (signature +2) ..... $\underline{\eta}$
   b. Universal time ..... t
   c. Scalar field ..... φ
   d. One-form field ..... $\underline{\psi}$
   e. Physical metric ..... $\underline{g}$

3. Gravitational field equations:

   a. Background metric is flat,

   $$\underline{\text{Riemann}}(\underline{\eta}) = 0 \ .$$

   b. "Meshing" of $\underline{\eta}$, t, $\underline{\psi}$:

   $$t_{|\alpha\beta} = 0 \ ,$$

   $$t_{|\alpha} t_{|\beta} \eta^{\alpha\beta} = -1 \ ,$$

   $$t_{|\alpha} \psi_{\beta} \eta^{\alpha\beta} = 0 \ ,$$

   where "$_|$" denotes covariant derivative with respect to $\underline{\eta}$, and $\| \eta^{ij} \|$ is the inverse of $\| \eta_{ij} \|$.

   c. $\underline{g} = f_2(\varphi)\underline{\eta} + [f_2(\varphi) - f_1(\varphi)]\, \underline{dt} \otimes \underline{dt} - \underline{\psi} \otimes \underline{dt} - \underline{dt} \otimes \underline{\psi}$.
   Here $f_1(\varphi)$ and $f_2(\varphi)$ are arbitrary functions to be determined finally by experiment.

   d. Field equations for φ and $\underline{\psi}$ follow from the action principle

   $$\delta \int \mathfrak{L}\, d^4x = 0 \ , \text{ where } \mathfrak{L} = \mathfrak{L}_{NG} + \mathfrak{L}_G \ ,$$

   $$\mathfrak{L}_G = -\frac{1}{8\pi} \left\{ \frac{1}{e}\, \psi_{\alpha|\gamma}\, \psi_{\beta|\delta}\, \eta^{\alpha\beta}\, \eta^{\gamma\delta} - \varphi_{|\alpha}\, \varphi_{|\beta}\, \eta^{\alpha\beta} + [f_3(\varphi) + 1][\varphi_{|\alpha} t_{|\beta}\, \eta^{\alpha\beta}]^2 \right\} \sqrt{-\eta} \ ;$$

   e is a constant to be determined by experiment, $\mathfrak{L}_{NG} = L_{NG}\sqrt{-g}$, and L_{NG} is the standard Lagrangian density of special relativity with the metric of special relativity replaced by $\underline{g}$.

4. Influence of gravity on matter:

   Governed by action principle

   $$\delta \int \mathfrak{L}_M\, d^4x = 0 \ ,$$

   where particle world lines and nongravitational fields are varied.

equivalence class of all its representations. Tables I–IV present particular representations for the theories described there.

Spacetime theory. A "spacetime theory" is any theory that possesses a mathematical representation constructed from a 4-dimensional spacetime manifold and from geometric objects defined on that manifold. (For the definition of "geometric object," see Sec. 4.13 of Trautman.[10]) Henceforth we shall restrict ourselves to spacetime theories and to the above type of mathematical representations. The geometric objects of a particular representation will be called its *variables*; the equations which the variables must satisfy will be called the *physical laws* of the representation. [Example – general relativity (Table II): The physical laws are the Einstein field equations, Maxwell's equations, the Lorentz force law, etc.] [Example – Belinfante-Swihart theory (Table IV): The physical laws are $\underline{\text{Riemann}}(\underline{\eta}) = 0$, and the Euler-Lagrange equations that follow from $\delta \int \mathfrak{L} d^4x = 0$.]

Manifold mapping group (MMG). The MMG is the group of all diffeomorphisms of the spacetime manifold onto itself. Each diffeomorphism h, together with an initial coordinate system $x^\alpha(\mathcal{P})$, produces a new coordinate system

$$x^{\alpha\prime}(\mathcal{P}) = x^\alpha(h^{-1}\mathcal{P}) \ . \tag{1}$$

(Events are denoted by capital script letters.)

Kinematically possible trajectory (kpt). Consider a given mathematical representation of a given

TABLE IV. Belinfante-Swihart theory.

1. References: Summary and analysis of the theory by Lee and Lightman[14]; original paper by Belinfante and Swihart.[15]

2. Gravitational fields:

   a. Metric ..... $\underline{\eta}$
   b. Symmetric second-rank tensor ..... $\underline{h}$

3. Nongravitational variables:

   a. Electromagnetic vector potential ..... $\underline{A}$
   b. Electromagnetic field tensor (second-rank, antisymmetric) ..... $\underline{H}$
   c. World line of particle J, parametrized in an arbitrary manner ..... $z_J^\alpha(\lambda_J)$ [in a given coordinate system, world line is $x^\alpha = z_J^\alpha(\lambda_J)$].
   d. Velocity vector of particle J (defined along world line) ..... $\underline{a}_J(\lambda_J)$
   e. Momentum vector of particle J (defined along world line) ..... $\underline{\pi}_J(\lambda_J)$

4. Gravitational field equations:

   a. Metric is flat: $\underline{\text{Riemann}}(\eta) = 0$.
   b. Field equation for $\underline{h}$ follows from varying $h_{\alpha\beta}$ in $\delta \int \mathfrak{L} d^4x = 0$, where $\mathfrak{L}$ is given below.

5. Influence of gravity on matter:

   Equations for $\underline{A}$, $\underline{H}$, $\underline{z}_J$, $\underline{a}_J$, $\underline{\pi}_J$ follow from varying these quantities in $\int \mathfrak{L} d^4x = 0$.

6. Lagrangian density:

   a. $\mathfrak{L} = \mathfrak{L}_G + \mathfrak{L}_{NG}$.
   b. $\mathfrak{L}_G = -(1/16\pi)\eta^{\alpha\beta}\eta^{\lambda\mu}\eta^{\rho\sigma}(a h_{\lambda\rho|\alpha} h_{\mu\sigma|\beta} + f h_{\lambda\mu|\alpha} h_{\rho\sigma|\beta})(-\eta)^{1/2}$, where "$|$" denotes covariant derivative with respect to $\underline{\eta}$; a and f are constants to be determined by experiment, and $\eta \equiv \det \| \eta_{ij} \|$.
   c. $\mathfrak{L}_{NG} = (1/4\pi)(\tfrac{1}{4} H^{\nu\mu} H_{\nu\mu} - H^{\nu\mu} A_{\mu|\nu})(-\eta)^{1/2}$

$$+\sum_J \int_{-\infty}^{+\infty} [-m_J b_J + (\pi_{J\mu} - e_J A_\mu)\dot{z}_J^\mu - \pi_{J\mu} a_J^\mu]\delta^4[\underline{x} - \underline{z}_J(\lambda_J)] d\lambda_J$$

$$+\tfrac{1}{2} T^{\mu\nu} h_{\mu\nu} + K \sum_J \int_{-\infty}^{+\infty} m_J b_J \eta^{\alpha\beta} h_{\alpha\beta} \delta^4[\underline{x} - \underline{z}_J(\lambda_J)] d\lambda_J \ .$$

   d. Here e_J and m_J are the charge and rest mass of particle J; $\dot{z}_J^\mu \equiv dz_J^\mu/d\lambda_J$; $b_J \equiv (-a_J^\alpha a_{J\alpha})^{1/2}$; K is a constant to be determined by experiment; indices are raised and lowered with $\eta_{\alpha\beta}$; and

$$T^{\mu\nu} \equiv (1/4\pi)(H^{\lambda\mu} H_\lambda{}^\nu - \tfrac{1}{4}\eta^{\mu\nu} H^{\alpha\beta} H_{\alpha\beta})$$

$$+\sum_J \int_{-\infty}^{+\infty} a_J^\mu \pi_J^\nu \delta^4[\underline{x} - \underline{z}_J(\lambda_J)] d\lambda_J \ .$$

   e. In the action principle one varies $h_{\mu\nu}$, A_μ, $H_{\mu\nu}$, $z_J^\alpha(\lambda_J)$, $a_J(\lambda_J)$, $\pi_J(\lambda_J)$ independently; but one holds $\eta_{\mu\nu}$ fixed.

spacetime theory. A kpt of that representation is any set of values for the components of all the variables in any coordinate system. A kpt need not satisfy the physical laws of the representation. (Example – general relativity (Table II): A kpt is any set of functions $\{g_{\alpha\beta}(x) = g_{\beta\alpha}(x);\ F_{\alpha\beta}(x) = -F_{\beta\alpha}(x);\ z_k^\alpha(\tau_k); \ldots\}$ in any coordinate system, which – if they were to satisfy the physical laws – would represent metric, electromagnetic field, particle world lines, etc.) (Example – Belinfante-Swihart theory (Table IV): A kpt is any set of functions $\{\eta_{\alpha\beta}(x) = \eta_{\beta\alpha}(x),\ h_{\alpha\beta}(x) = h_{\beta\alpha}(x),\ A_\alpha(x), H_{\alpha\beta}(x) = -H_{\beta\alpha}(x),\ z_J^\alpha(\lambda_J),\ a_J^\alpha(\lambda_J),\ \pi_J^\alpha(\lambda_J)\}$ in any coordinate system.)

Dynamically possible trajectory (dpt). A dpt is any kpt that satisfies all the physical laws of the representation.

Covariance group of a representation. A group $\mathcal{G}$ is a covariance group of a representation if (i) $\mathcal{G}$ maps kpt of that representation into kpt; (ii) the kpt constitute "the basis of a faithful realization of $\mathcal{G}$" (i.e., no two elements of $\mathcal{G}$ produce identical mappings of the kpt)[19]; (iii) $\mathcal{G}$ maps dpt into dpt. (Example – MMG is a covariance group of each of the representations of theories in Tables I–IV.) (Example – Electromagnetic gauge transformations, $A_\mu \to A_\mu + \varphi_{,\mu}$, are a covariance group of the representation of Belinfante-Swihart theory given in Table IV.) By *complete covariance group* we shall mean the largest covariance group of the

representation. By *generally covariant representation* of a theory we shall mean any representation for which MMG is a covariance group. (An argument due to Kretschmann[20] shows that every spacetime theory possesses generally covariant representations.) By *internal covariance group* we shall mean a covariance group that involves *no* diffeomorphisms of spacetime onto itself. (Example – Electromagnetic gauge transformations are an internal covariance group.) By *external covariance group* we shall mean a covariance group that is a subgroup of MMG. The complete covariance group of a representation need not be the direct product of its complete (i.e., largest) internal covariance group with its complete external covariance group. It may also include transformations that are "partially internal" and "partially external" and cannot be split up. [Example – When one formulates Newton-Cartan theory in a Galilean coordinate representation (see the Appendix, which should not be read until one has finished this entire section), one obtains a complete covariance group described by Eqs. (A5). The complete external covariance group consists of (A5a) and (A5b). There is no internal covariance group. The transformations (A5c) are mixed internal-external transformations that belong to the complete covariance group.]

We shall use the following notation to describe a particular element G of the covariance group, and its effect. G consists of a diffeomorphism h [Eq. (1), above] and an internal transformation H:

$$G = (h, H). \tag{2}$$

If G is an external transformation (element of MMG), then H must be the identity operation; if G is an internal transformation, then h is the identity mapping; if G is a mixed internal-external transformation, then neither h nor H is an identity. Denote the variables of the representation (geometric objects) by y, and their components at a point $\mathcal{P}$ in a coordinate system $\{x^\alpha\}$ by $y_A(\mathcal{P}, \{x^\alpha\})$. The set of functions

$$y_A(\mathcal{P}, \{x^\alpha\}), \quad \mathcal{P} \text{ varying and } \{x^\alpha\} \text{ fixed} \tag{3}$$

constitute a kpt. The diffeomorphism h maps this kpt into $y_A(\mathcal{P}, \{x^{\alpha'}\})$, where $\{x^{\alpha'}\}$ is the coordinate system of Eq. (1). The internal transformation H converts y into a new geometric object,

$$y' \equiv Hy. \tag{4}$$

The net effect of G on the kpt (3) is

$$G: \quad y_A(\mathcal{P}, \{x^\alpha\}) \to y'_A(\mathcal{P}, \{x^{\alpha'}\}). \tag{5}$$

It is often useful to characterize G by the functions

$$\begin{aligned}\bar{\delta} y_A(\mathcal{P}, \{x^{\alpha'}\}) &\equiv y'_A(\mathcal{P}, \{x^{\alpha'}\}) - y_A(h^{-1}\mathcal{P}, \{x^\alpha\}) \\ &= y'_A\big|_{\text{evaluated at } x^{\alpha'}(\mathcal{P})} \\ &\quad - y_A\big|_{\text{evaluated at } x^\alpha = x^{\alpha'}(\mathcal{P})}.\end{aligned} \tag{6}$$

Note that these "changes in y" satisfy the relation

$$\bar{\delta}(y_{A,\mu})(\mathcal{P}, \{x^{\alpha'}\}) = [\bar{\delta} y_A(\mathcal{P}, \{x^{\alpha'}\})]_{,\mu'}, \tag{7}$$

where a comma denotes partial derivative, and also the relation

$$\bar{\delta} y_A = (Hy)_A(\mathcal{P}, \{x^{\alpha'}\}) - (hy)_A(\mathcal{P}, \{x^{\alpha'}\}), \tag{8}$$

where hy is the geometric object obtained by "dragging along with h" (see p. 86 of Trautman[10]).

Of particular interest are the infinitesimal elements of a covariance group. [From them one can generate that topologically connected component[21] of the group which contains the identity. The other connected components, if any, are typically obtained by bringing into play a discrete set of group elements (space reflections, time inversions, etc.).] Let $G_\epsilon = (h_\epsilon, H_\epsilon)$ be a one-parameter family of elements (curve in group space parametrized by ϵ), with G_0 the identity. Denote by $\underline{\xi}$ the infinitesimal generator of the diffeomorphism h_ϵ:

$$\underline{\xi} \equiv [d(h_\epsilon \mathcal{P})/d\epsilon]_{\epsilon=0}. \tag{9}$$

Then, to first order in ϵ, Eq. (8) reduces to

$$\bar{\delta} y_A(\mathcal{P}, \{x^\alpha\}) = \epsilon \left\{ (\pounds_{\underline{\xi}} y)_A(\mathcal{P}, \{x^\alpha\}) + \left[\frac{d}{d\epsilon} (H_\epsilon y)_A(\mathcal{P}, \{x^\alpha\}) \right]_{\epsilon=0} \right\}, \tag{10}$$

where $\pounds_{\underline{\xi}}$ is the Lie derivative along $\underline{\xi}$ (Sec. 4.15 of Trautman[10]).

Equivalence classes of dpt. Two dpt are members of the same equivalence class if one of them is mapped into the other by some element of the complete covariance group. (Example – When MMG is a covariance group, all dpt that are obtained from each other by coordinate transformations belong to the same equivalence class.) If a generally covariant representation possesses no internal covariance groups, then there is a one-to-one correspondence between equivalence classes of dpt and the geometric, coordinate-independent solutions of its geometric, coordinate-independent physical laws.

Confined, absolute, and dynamical variables. The variables of a generally covariant representation split up into three groups: "confined variables," "absolute variables," and "dynamical variables." The confined variables are those which do *not* constitute the basis of a faithful realization of MMG. (Examples – All universal constants, such

as the charge of the electron, are confined variables. The world line of a particle is not a confined variable, as one sees by this procedure: (i) Characterize the world line by the scalar field

$$\tau(\mathcal{P}) = \begin{cases} 0, & \text{if } \mathcal{P} \text{ is } \textit{not} \text{ on world line;} \\ \text{proper time of particle,} & \\ \quad \text{if } \mathcal{P} \textit{ is} \text{ on world line.} \end{cases} \tag{11}$$

(ii) Verify that an element of MMG can be characterized uniquely by the manner in which it maps the set of all kinematically possible world lines [all functions $\tau(x^\alpha)$ that are zero everywhere except along a curve, and are monotonic along that curve] into each other. (iii) Thereby conclude that a particle world line does constitute the basis for a faithful realization of MMG, and therefore that it is not a confined variable.) To determine whether an unconfined variable B is absolute or dynamical, perform the following test: Pick out an arbitrary dpt, and let $\overline{B}_A(x^\alpha)$ be the functions which describe the components of B for that dpt. Then examine each equivalence class of dpt to see whether these same functions $\overline{B}_A$ appear somewhere in it. If they do, for every equivalence class and for every choice of the arbitrary initial dpt, then B is an *absolute variable*. If they do not, for some particular choice of the initial dpt and for some particular equivalence class, then B is a *dynamical variable*. Some dynamical variables contain absolute parts, and some dynamical and absolute variables contain confined parts. [Example – Belinfante-Swihart theory (Table IV): $\eta_{\alpha\beta}$ is an absolute variable; $h_{\alpha\beta}$ and all the nongravitational variables are dynamical.] [Example – Ni's theory (Table III): $\underline{\eta}$ and t are absolute variables; $\underline{\psi}$, φ, and $\underline{g}$ are dynamical. Although $\underline{\psi}$ is dynamical, it contains an absolute part – the projection of $\underline{\psi}$ on $\underline{dt}$ (i.e., $\psi_\alpha t_{|\beta}\eta^{\alpha\beta}$). The remaining, "spatial" part of $\underline{\psi}(\underline{\psi} + \psi_\alpha t_{|\beta}\eta^{\alpha\beta}\underline{dt})$ is fully dynamical. Although t is absolute, it contains a confined part – its "origin," or equivalently, its value at some fixed fiducial event $\mathcal{P}_0$. One can remove this confined part from t by passing from t to the 1-form field $\underline{dt}$.] [Example – general relativity (Table II): All the unconfined variables are dynamical, and they contain no absolute parts. It is this feature that distinguishes general relativity from almost all other theories of gravity (see JLA[11]; also Chap. 17 of MTW, where absolute variables are called "prior geometry").] (Example – Newton-Cartan theory: In the representation of Table I, t and $\underline{\gamma}$ are absolute variables; $\underline{\nabla}$ is dynamical. As in Ni's theory, the origin of t is a confined variable and can be split off by passing from t to $\underline{dt}$. Although the covariant derivative $\underline{\nabla}$ is dynamical, it contains absolute parts. A decomposition of $\underline{\nabla}$ into its absolute and dynamical parts is performed in the Appendix [Eq. (A1e)]. After that decomposition the theory takes on a new mathematical representation with absolute variables $\underline{\beta}$, $\underline{\gamma}$, $\underline{D}$, and dynamical variables Φ and $\underline{\nabla}$.)

Irrelevant variables. A set of variables of a generally covariant representation is called irrelevant if (i) its variables are not coupled by the physical laws to the remaining variables of the representation, and (ii) its variables can be eliminated from the representation without altering the structure of the equivalence classes of dpt and without destroying general covariance. A variable that is not irrelevant is called "relevant." Some variables contain both relevant and irrelevant parts. (Example – The gauge of the electromagnetic vector potential is irrelevant. So is any other variable that can be forced to take on any desired set of values by imposing an appropriate internal covariance transformation.) [Example – In Ni's theory (Table IV) and the Newton-Cartan theory (Table I) the origin of universal time t is an irrelevant variable.]

Fully reduced, generally covariant representation. A generally covariant representation is called "fully reduced" if (i) it contains no irrelevant variables, (ii) its dynamical variables contain no absolute parts, and (iii) its dynamical and absolute variables contain no confined parts. [Example – Newton-Cartan theory: The representation of Table I is generally covariant, but not fully reduced. To reduce it one must follow the procedure of the Appendix: (i) Remove the irrelevant origin of t by passing from t to $\underline{\beta} = \underline{dt}$; (ii) split $\underline{\nabla}$ into its absolute and dynamical parts. The resulting representation is not quite fully reduced because it possesses the internal covariance transformation (A3′a) with an associated, irrelevant "gauge arbitrariness" in $\underline{D}$ and Φ. When one removes that irrelevance by fixing the "gauge" once and for all (e.g., by requiring, for an island universe, that $\left\{ {\alpha \atop \beta\gamma} \right\} = 0$ in any Galilean frame where the total 3-momentum vanishes), then one obtains a fully reduced representation.]

Boundary conditions, prior geometric constraints, decomposition equations, and dynamical laws. In a given mathematical representation of a given theory, the physical laws break up into four sets: (i) boundary conditions – those laws which involve only confined variables; (ii) prior geometric constraints[22] – those which involve absolute variables and possibly also confined variables, but not dynamical variables; (iii) decomposition equations – those which express a dynamical variable algebraically in terms of other variables; (iv) dynamical laws – all others. [Example – Ni's theory (Table III): Equations (3a) and (3b) are prior geometric constraints; Eq. (3c)

is a decomposition equation; and the equations that follow from the variational principle are all dynamical. If one augments the theory by cosmological demands that $\underline{\psi}$ and φ go to zero at spatial infinity, those demands are boundary conditions.] [Example – general relativity (Table II): All physical laws are dynamical.] [Example – Belinfante-Swihart theory (Table IV): Riemann ($\underline{\eta}$) = 0 is a prior geometric constraint; the equations obtained from the variational principle are dynamical.] [Example – Newton-Cartan theory (Table I): In the mathematical formulation of Table I, Eqs. (3a)–(3d) are all dynamical laws. One has the feeling, however, that they ought not to be dynamical, because they involve only gravitational fields; they make no reference to any source of gravity. Only (3e) contains a source, so only it "ought to be" dynamical. The failure of one's "ought-to" intuition results from one's failure to split $\underline{\nabla}$ up into its absolute and dynamical pieces. Such a split (see Appendix) results in a new mathematical formulation of the theory, with just one dynamical gravitational law: (A1f), which is equivalent to (3e) of Table I. Of the other gravitational equations in the new formulation, (A1a)–(A1d) are prior geometric constraints, and (A1e) is a decomposition equation.]

Symmetry group. Let G be an element of the complete covariance group of a representation. Examine the change produced by G in every variable B that (i) is absolute, and (ii) has had all irrelevant, confined parts removed from itself. If

$$\bar{\delta} B_A(\mathcal{P}, \{x^{\alpha'}\}) = 0 \text{ at all } \mathcal{P} \text{ and for all coordinate systems } \{x^{\alpha'}\} \tag{12}$$

for every such B, then G is called a *symmetry transformation*. Any group of symmetry transformations is called a *symmetry group*; the largest group of symmetry transformations is called the *complete symmetry group* of the representation. [Note: That component of the complete symmetry group which is topologically connected to the identity is generated by infinitesimal transformations. One can find all the infinitesimal generators by solving Eqs. (10) and (12) for $\underline{\xi}$, and for $(dH_\epsilon/d\epsilon)_{\epsilon=0}$.] [Another note: If the absolute variables B are all tensor or affine-connection fields, then $\bar{\delta} B$ are all tensor fields, so

$$(\bar{\delta} B_A = 0 \text{ for all } \mathcal{P} \text{ in one coordinate system}) \Rightarrow (\bar{\delta} B_A = 0 \text{ for all } \mathcal{P} \text{ in every coordinate system}). \tag{13}$$

Hence, in this case one can confine attention to any desired, special coordinate system when testing for symmetry transformations.] [Example – Belinfante-Swihart theory (Table IV): The complete symmetry group consists of the Poincaré group (inhomogeneous Lorentz transformations) together with the electromagnetic gauge transformations. One proves this most easily in a global Lorentz frame of $\underline{\eta}$; one can restrict calculations to this frame because the absolute variable $\underline{\eta}$ is a tensor.] [Example – Ni's theory (Table III): Symmetry transformations are analyzed most easily in a coordinate system where $x^0 = t =$ (universal time), and $\eta_{\alpha\beta}$ has the Minkowski form. Any symmetry transformation must leave $\bar{\delta}\eta_{\alpha\beta} = \bar{\delta} t_{,\alpha} = \bar{\delta}(\eta^{\alpha\beta} t_{,\alpha} \psi_\beta) = 0$. Thus, the symmetry transformations are (i) electromagnetic gauge transformations; (ii) spacetime translations, $x^{\alpha'} = x^\alpha + a^\alpha$ with a^α a constant; (iii) time-independent spatial rotations, $x^{0'} = x^0$ and $x^{j'} = R^{jk} x^k$ with $\|R^{jk}\|$ a rotation matrix; (iv) spatial reflections.] [Example – general relativity (Table II): There are no absolute variables, so the complete covariance group and the complete symmetry group are identical; they are the MMG plus electromagnetic gauge transformations.] (Example – Newton-Cartan theory: See Appendix.) An *external symmetry group* is a symmetry group that is a subgroup of MMG. An *internal symmetry group* is a symmetry group that involves no diffeomorphisms of spacetime onto itself. The complete symmetry group need not be the direct product of the external symmetries and the internal symmetries; it may also include symmetries that are partially internal and partially external and cannot be split up. [Example – Newton-Cartan theory in the representation of the Appendix: Transformations (A5c) are partially internal and partially external.]

III. GRAVITATION THEORIES AND EQUIVALENCE PRINCIPLES

We now turn from general spacetime theories to the special case of gravitation theories. We cannot discuss gravitation theories without making somewhat precise the distinction between gravitational phenomena and nongravitational phenomena. There seem to be a variety of ways in which one might make this distinction. Somewhat arbitrarily, but after considerable thought, we have chosen to regard as "gravitational" those phenomena which either are absolute or "go away" as the amount of mass-energy in the experimental laboratory decreases. In other words, gravitational phenomena are either prior geometric effects or effects generated by mass-energy. This means that the flat background metric $\underline{\eta}$ of Belinfante-Swihart theory is a gravitational field; the metric

of general relativity is a gravitational field; but the torsion of Cartan's modified general relativity,[23] which is generated by spin rather than by mass-energy, is not a gravitational field.

We try to make the above statements more precise by introducing the following concepts.

Local test experiment. A "local test experiment" is any experiment, performed anywhere in spacetime, in the following manner. A shield is set up around the experimental laboratory. When analyzed using the concepts and experiments of special relativity, this shield must have arbitrarily small mass-energy and must be impermeable to electromagnetic fields, to neutrino fields, and to real (as opposed to virtual) particles. The experiment is performed, with freely falling apparatus, in the center of the shielded laboratory, in a region so small that inhomogeneities in all external fields are unimportant. One makes sure that external inhomogeneities are unimportant by performing a sequence of experiments of successively smaller size (with size of shield and external conditions unchanged), until the experimental result approaches a constant value asymptotically. (Examples – The experiment might be a local measurement of the electromagnetic fine-structure constant, or a Cavendish experiment with two lead spheres, or a series of Cavendish experiments involving lead spheres and small black holes.)

Local, nongravitational, test experiment. A "local, nongravitational test experiment" is a local test experiment with these properties: (i) When analyzed in the center-of-mass Galilean frame, using the Newtonian theory of gravity, and using all forms of special relativistic mass-energy as sources for the Newtonian potential Φ, the matter and fields inside the shield must produce a Φ with

$$|\Phi \text{ (at any point inside shield)} - \Phi \text{ (at any point on shield)}| \ll 1 .$$

(ii) When the experiment is repeated, with successively smaller mass-energies inside the shield (as deduced using special relativity theory) – but leaving unchanged the characteristic sizes, intrinsic angular momenta, velocities, and charges (electric, baryonic, leptonic, etc.) of its various parts – the experimental result does not change. (Examples: A measurement of the electromagnetic fine-structure constant *is* a local, nongravitational test experiment; a Cavendish experiment is not.)

Gravitation theory. A "gravitation theory," or "theory of gravity," is any space-time theory which correctly predicts Kepler's laws for a binary star system that (i) is isolated in interstellar space ("local test experiment"); (ii) consists of two "normal stars" (stars with $|\Phi| \ll 1$ throughout their interiors); and (iii) has periastron p large compared to the stellar radii, $p \gg R$. The theory's predictions must not deviate from Kepler's laws by fractional amounts exceeding the larger of $|\Phi|_{\max}$ and p/R. (Note: To agree with experiment in the solar system, the theory will have to reproduce Kepler much more accurately than this.) (Examples – The theories in Tables I–IV are all gravitation theories.)

In the absence of gravity. The phrase "in the absence of gravity" means "when analyzing any local, nongravitational test experiment for which the shield is spherical, has arbitrarily large radius, and is surrounded by a spherically symmetric sea of matter." *"To turn off gravity"* means "to pass from a generic situation to a situation where gravity is absent." *"To turn on gravity"* means "to pass from a situation where gravity is absent to a generic situation."

Gravitational field. In a given representation of a given gravitation theory, any unconfined, relevant variable B is a "gravitational field" if, in the absence of gravity, it reduces to a constant, or to an absolute variable, or to an irrelevant variable. In particular, every absolute, relevant variable is a gravitational field. [Example – general relativity (Table II): For local, nongravitational test experiments, analyzed using Fermi-normal coordinates, one gets the same result whether one uses the correct $\underline{g}$ or one replaces it by a flat Minkowski metric $\underline{\eta}$ (absolute variable). Thus $\underline{g}$ is a gravitational field.] [Example – Newton-Cartan theory (Table I): t and $\underline{\gamma}$ are already absolute, so they are gravitational fields; $\underline{\nabla}$ can be replaced by the Riemann-flat $\underline{D}$ of the Appendix without affecting local, nongravitational experiments, so it is also a gravitational field.] (Example – Cartan's modification of general relativity, with torsion[23]: The torsion is generated by spin. Therefore, it must remain a dynamical variable in analyses of local, nongravitational test experiments. It is not a gravitational field.)

Dicke's[24] weak equivalence principle (WEP).[25] The weak equivalence principle states: *If an uncharged test body is placed at an initial event in spacetime, and is given an initial velocity there, then its subsequent world line will be independent of its internal structure and composition.* Here by "uncharged test body" is meant an object (i) that is shielded, in the sense used above in defining "local test experiments"; (ii) that has negligible self-gravitational energy, when analyzed using Newtonian theory; (iii) that is small enough in size so its coupling (via spin and multipole moments) to inhomogeneities of external fields can be ig-

nored. These constraints guarantee that any test of WEP is a local, nongravitational test experiment.

WEP is called "universality of free fall" by MTW,[12] and is called "equality of passive and inertial masses" by Bondi.[26]

The experiments of Eötvös *et al.*,[6] Dicke *et al.*,[7] and Braginsky *et al.*[8] are direct tests of WEP. Braginsky, whose experiment is the most recent, reports that the relative acceleration of an aluminum test body and a platinum test body placed in the sun's gravitational field at the location of the earth's orbit is

$$\text{(relative acceleration)} < 0.9\times10^{-12}(GM_\odot/r_{\text{orbit}}^2) = 0.5\times10^{-12}\ \text{cm/sec}^2 \quad \text{(95\% confidence)}.$$

If WEP is correct, then the world lines of test bodies are a preferred family of curves (without parametrization) filling spacetime – with a single unique curve passing in each given direction through each given event. But WEP does *not* guarantee that these curves can be regarded as geodesics of the spacetime manifold; only if these curves have certain special properties can they be geodesics.[27]

Einstein equivalence principle (EEP). The Einstein equivalence principle states that (i) WEP is valid, and (ii) the outcome of any local, nongravitational test experiment is independent of where and when in the universe it is performed, and independent of the velocity of the (freely falling) apparatus. (Example – Dimensionless ratios of nongravitational physical constants must be independent of location, time, and velocity.) The experimental evidence supporting EEP is reviewed in Secs. 38.5 and 38.6 of MTW.[12]

Dicke's[24] strong equivalence principle (SEP). SEP states that (i) WEP is valid, and (ii) the outcome of any local test experiment – gravitational or nongravitational – is independent of where and when in the universe it is performed, and independent of the velocity of the (freely falling) apparatus. (Example – The Dicke-Brans-Jordan theory, with its variable "gravitational constant" as measured by Cavendish experiments, satisfies EEP but violates SEP.)

Two types of effects can lead to a breakdown of SEP: "preferred-location effects" and "preferred-frame effects." Perform a local test experiment, gravitational or nongravitational. If the experimental result depends on the location of the freely falling experimenter, but not on his velocity there, the phenomenon being measured is called a *preferred-location effect.* If it depends on the velocity of the experimenter, it is called a *preferred-frame effect.*[28] [Examples – A cosmological time variation in the "gravitational constant" (as measured by Cavendish experiments) is a preferred-location effect. Anomalies in the earth's tides and rotation rate due to the orbital motion of the earth around the sun and the sun through the galaxy[28] are preferred-frame effects.]

A theory of gravity obeys SEP if and only if it obeys EEP, and it possesses no preferred-frame or preferred-location effects.

Any theory for which the complete external symmetry group excludes boosts will presumably exhibit preferred-frame effects. But preferred-frame effects can also show up when boosts are in the symmetry group. (Example – The vector-tensor theory of Nordtvedt, Hellings, and Will[28] exhibits preferred-frame effects but possesses MMG as a symmetry group.) For further discussion see "*metric theory of gravity*," below.

IV. PROPERTIES AND CLASSES OF GRAVITATION THEORIES

Completeness of a theory. A gravitation theory is "complete" if it makes a definite prediction (not necessarily the correct prediction) for the outcome of any experiment that current technology is capable of performing. (Standard quantum-mechanical limitations on the definiteness of the prediction are allowed.) To be complete, the theory must predict results for nongravitational experiments as well as for gravitational experiments. Of course, it can do so only if it meshes with and incorporates (perhaps in modified form) all the nongravitational laws of physics. If a theory is complete so far as all "classical" experiments are concerned, but has not yet been meshed with the quantum-mechanical laws of physics, we shall call it *classically complete.*

Self-consistency of a theory. A gravitation theory is "self-consistent" if its prediction for the outcome of every experiment is unique – i.e., if, when one calculates the prediction by different methods, one always gets the same result.

Reference 2 discusses completeness and self-consistency in greater detail, and gives examples of incomplete theories and self-inconsistent theories.

Relativistic theory of gravity. A theory of gravity is "relativistic" if it possesses a representation ("relativistic representation") in which, in the absence of gravity, the physical laws reduce to the standard laws of special relativity. (Examples – General relativity, Ni's theory, and the Belinfante-Swihart theory are relativistic; the Newton-Cartan theory is not, nor is Cartan's torsion-endowed modification of general relativity.[23])

Metric theory of gravity. By "metric theory" we mean any theory that possesses a mathematical representation ("metric representation") in which (i) spacetime is endowed with a metric; (ii) the world lines of test bodies are the geodesics of that metric; and (iii) EEP is satisfied, with the nongravitational laws in any freely falling frame reducing to the laws of special relativity.[29] Any theory or representation that is not metric will be called *nonmetric*. [Examples – General relativity and Ni's theory are metric theories, and the representations given in Tables II and III are metric; the Belinfante-Swihart theory is nonmetric,[14] but can be made metric by suitable modifications.[14,30] The Newton-Cartan theory is nonmetric. The Dicke-Brans-Jordan theory is metric; the representation of Ref. 16 is a metric representation; the representation of Ref. 18 ("conformally transformed representation"; "rubber meter sticks") is nonmetric.]

In any metric theory, the metric that enters into EEP is called the "*physical metric*." All other gravitational fields are called "*auxiliary gravitational fields*." Relevant auxiliary scalar fields typically produce preferred-location effects; other relevant auxiliary gravitational fields (vector, tensor, etc.) typically produce preferred-frame effects. This is true independently of whether or not the auxiliary fields are absolute variables or are dynamical – i.e., independently of whether the complete external symmetry group is MMG or is more restrictive.

Clearly, every metric theory is relativistic, but relativistic theories need not be metric [example: the Belinfante-Swihart theory]. Ni[31] has given a partial catalog of metric theories. Will and Nordtvedt[32] have developed a "parametrized post-Newtonian formalism" for comparing metric theories with each other and with experiment.

Prior geometric theories. Any gravitation theory will be called a "prior geometric theory" if it possesses a fully reduced, generally covariant representation that contains absolute variables. (Examples – The Newton-Cartan theory, Ni's theory, and the Belinfante-Swihart theory are prior geometric; general relativity and the Dicke-Brans-Jordan theory are not.)

Lorentz-symmetric representations and theories. A generally covariant representation is called "Lorentz symmetric" if its complete external symmetry group is the Poincaré group – with or without inversions and time reversal. We suspect that, for any theory, all fully reduced, generally covariant representations must have the same complete external symmetry group. Assuming so, we define a theory to be "Lorentz symmetric" if its fully reduced, generally covariant representations are Lorentz symmetric. (Example – General relativity is not Lorentz symmetric; the complete external symmetry group of its fully reduced, standard representation is too big – it is MMG rather than Poincaré.) (Example – Ni's theory is not Lorentz symmetric; as with the Newton-Cartan theory, the complete external symmetry group is too small.) (Example – Belinfante-Swihart theory is Lorentz symmetric.)

Elsewhere in the literature one sometimes finds Lorentz-symmetric theories called "Lorentz-invariant theories" or "flat-space theories."

Lagrangian-based representations and theories. A generally covariant representation of a spacetime theory is called Lagrangian-based if (i) there exists an action principle that is extremized with respect to variations of all dynamical variables but *not* with respect to variations of absolute or confined variables, and (ii) from the action principle follow all the dynamical laws but none of the other physical laws. The issue of whether the other physical laws (boundary conditions, decomposition equations, and prior geometric constraints) are imposed before the variation or afterwards does not affect the issue of whether the representation is Lagrangian-based. A theory is called Lagrangian-based if it possesses a generally covariant, Lagrangian-based representation. (Examples – General relativity, Ni's theory, and the Belinfante-Swihart theory are all Lagrangian-based.)

The Lagrangian density $\mathfrak{L}$ of a Lagrangian-based representation (which appears in the action principle in the form $\delta\int\mathfrak{L}d^4x=0$) can be split up into two parts: $\mathfrak{L}=\mathfrak{L}_G+\mathfrak{L}_{NG}$. The *gravitational part* $\mathfrak{L}_G$ is the largest part that contains only gravitational fields. The *nongravitational part* $\mathfrak{L}_{NG}$ is the rest.

V. UNIVERSAL COUPLING

We turn attention, now, from our glossary of concepts to some applications. We begin in this section by analyzing the overlap between metric theories and relativistic, Lagrangian-based theories.

As motivation for the analysis, consider any relativistic representation of a relativistic theory of gravity. In the absence of gravity that representation reduces to special relativity – so, in particular, it possesses a flat Minkowski metric $\eta_{\alpha\beta}$. By continuity one expects the representation to possess, in the presence of gravity, at least one second-rank, symmetric tensor gravitational field $\psi_{\alpha\beta}$ that reduces to $\eta_{\alpha\beta}$ as gravity is turned off. Indeed, this is the case for all relativistic theories with which we are familiar. (Example –

general relativity: The curved-space metric $g_{\alpha\beta}$ reduces to $\eta_{\alpha\beta}$ when gravity is turned off.) (Example – Ni's theory: There are a variety of second-rank, symmetric tensor gravitational fields that reduce to $\eta_{\alpha\beta}$. They include the flat background metric $\eta_{\alpha\beta}$, the physical metric $g_{\alpha\beta}$, any tensor field of the form $[1+f(\varphi)]\eta_{\alpha\beta}$, where $f(\varphi)$ is an arbitrary function with $f(0)=0$, etc.) [Example – Belinfante-Swihart theory: $\eta_{\alpha\beta}$, $\eta_{\alpha\beta}+h_{\alpha\beta}$, $\eta_{\alpha\beta}(1+3h_\mu{}^\mu)-17h_\alpha{}^\mu h_\mu{}^\beta$ all reduce to $\eta_{\alpha\beta}$ when gravity is turned off.]

Next consider any *Lagrangian-based, relativistic theory*. Being relativistic, it must possess a generally covariant, Lagrangian-based representation in which, as gravity is turned off, the nongravitational part of the Lagrangian $\mathfrak{L}_{NG}$ approaches the total Lagrangian of special relativity. Adopt that representation. Then, in the presence of gravity $\mathfrak{L}_{NG}$ will presumably contain at least one second-rank, symmetric, tensor gravitational field $\psi_{\alpha\beta}$ that reduces to $\eta_{\alpha\beta}$ as gravity is turned off. Roughly speaking, if $\mathfrak{L}_{NG}$ contains precisely one such $\psi_{\alpha\beta}$ and contains no other gravitational fields, then the theory is said to be "universally coupled."[33]

More precisely, we say that a Lagrangian-based, relativistic theory is *universally coupled* if it possesses a representation ("universally coupled representation") with the following properties: (i) The representation is generally covariant and Lagrangian-based. (ii) $\mathfrak{L}_{NG}$ contains precisely one gravitational field, and that field is a second-rank, symmetric tensor $\psi_{\alpha\beta}$ with signature +2 throughout spacetime. (iii) In the limit as gravity is turned off $\psi_{\alpha\beta}$ becomes a Riemann-flat second-rank, symmetric tensor field $\eta_{\alpha\beta}$; and whenever $\psi_{\alpha\beta}$ is replaced by such an $\eta_{\alpha\beta}$, $\mathfrak{L}_{NG}$ becomes the total Lagrangian of special relativity. (iv) The prediction for the result of any local, nongravitational experiment anywhere in the universe is unchanged when, throughout the laboratory, one replaces $\psi_{\alpha\beta}$ by a Riemann-flat second-rank, symmetric tensor.

The following theorem reveals the key role of universal coupling as a link between Lagrangian-based theories and metric theories: *Consider all Lagrangian-based, relativistic theories of gravity. Every such theory that is universally coupled is a metric theory; and, conversely, every metric theory in this class is universally coupled.*

Proof: Let $\mathfrak{A}$ be a Lagrangian-based, relativistic, universally coupled theory. Adopt a universally coupled representation. Use that representation to analyze any local, nongravitational test experiment anywhere in spacetime. Use the mathematical tools of Riemannian geometry, treating the unique gravitational field $\psi_{\alpha\beta}$ that appears in $\mathfrak{L}_{NG}$ as a metric tensor. In particular, introduce a Fermi-normal coordinate system ($\psi_{\alpha\beta}=\eta_{\alpha\beta}$, $\Gamma^\alpha{}_{\beta\gamma}=0$ at the center of mass of the laboratory). Condition (iv) for universal coupling guarantees that the predictions of the representation will be unchanged if we replace $\psi_{\alpha\beta}$ by $\eta_{\alpha\beta}$ throughout the laboratory. Do so. Then condition (iii) for universal coupling guarantees that $\mathfrak{L}_M$ is the total Lagrangian of special relativity. The dynamical laws that follow from

$$\delta\int(\mathfrak{L}_G+\mathfrak{L}_{NG})d^4x=0$$

by varying all nongravitational variables also follow from

$$\delta\int\mathfrak{L}_{NG}\,d^4x=0;$$

in this representation and coordinate system they are the laws of special relativity. Thus, the outcome of the local, nongravitational test experiment is governed by the standard laws of special relativity, irrespective of the location and velocity of the apparatus. This guarantees that theory $\mathfrak{A}$ is a metric theory.

Proof of converse: Let $\mathfrak{H}$ be a Lagrangian-based, metric theory. Adopt a Lagrangian-based, metric representation. Since all unconfined, nongravitational variables are dynamical, they must all be varied in $\delta\int\mathfrak{L}d^4x=0$. Moreover, since they appear in $\mathfrak{L}_{NG}$ but not in $\mathfrak{L}_G$, their Euler-Lagrange equations are obtained equally well from

$$\delta\int\mathfrak{L}_{NG}\,d^4x=0\,.$$

Call those Euler-Lagrange equations (obtained by varying all unconfined, nongravitational variables in $\delta\int\mathfrak{L}_{NG}d^4x=0$) the "nongravitational laws." Let a freely falling observer anywhere in spacetime, with any velocity, perform a local, nongravitational test experiment. Analyze that experiment in a local Lorentz frame of the physical metric $g_{\alpha\beta}$ using the above nongravitational laws. Because the theory is metric, the predictions must be the same as those of special relativity. Hence, the nongravitational laws – in *any* local Lorentz frame of $g_{\alpha\beta}$ *anywhere* in the universe – must reduce to the laws of special relativity. This is possible only if (i) those laws – and hence also $\mathfrak{L}_{NG}$ – contain no reference to any gravitational field except $g_{\alpha\beta}$,[34] and (ii) $\mathfrak{L}_{NG}$ is some version of the total special relativistic Lagrangian, with $\eta_{\alpha\beta}$ replaced by $g_{\alpha\beta}$. These properties of $\mathfrak{L}_{NG}$, plus the definition of "metric theory," guarantee directly that the four conditions for universal coupling are satisfied. Hence, theory $\mathfrak{H}$ is universally coupled. QED.

VI. SCHIFF'S CONJECTURE

Schiff's conjecture[5] *states that any complete and self-consistent gravitation theory that obeys WEP must also, unavoidably, obey EEP.*

General relativity is an example. It endows spacetime with a metric; it obeys WEP by predicting that all uncharged test bodies fall along geodesics of that metric, with each geodesic world line determined uniquely by an initial event and an initial velocity; it achieves completeness by demanding that in every local, freely falling frame the nongravitational laws of physics take on their standard special relativistic forms; and by this method of achieving completeness, it obeys EEP.

The Newton-Cartan theory is another example. It was complete and self-consistent within the framework of nineteenth century technology. It endows spacetime with an affine connection; it obeys WEP by predicting that all uncharged test bodies fall along geodesics of that affine connection, with each geodesic world line determined uniquely by an initial event and an initial velocity; it achieves completeness by demanding that in every local, freely falling frame the laws of physics take on their standard nongravitational Newtonian form; and by this method of achieving completeness, it obeys EEP.

Before accepting Schiff's conjecture as plausible, one should search the literature for a counterexample – i.e., for a theory of gravity which somehow achieves completeness, and somehow obeys WEP, but fails to obey EEP. Several Lagrangian-based theories which one finds in the literature might conceivably be counterexamples, but they have not been analyzed with sufficient care to allow any firm conclusion. Subsequent papers[9,14] will show that the most likely counterexample, Belinfante-Swihart theory, actually fails to satisfy WEP, violates the ED experimental results, and is thus not a counterexample at all.

One can make Schiff's conjecture seem very plausible within the framework of relativistic, Lagrangian-based theories (the case of greatest interest; see Sec. I) by the following line of argument.[35]

Consider a Lagrangian-based, relativistic theory, and ask what constraints WEP places on the Lagrangian. WEP probably forces $\mathfrak{L}_{NG}$ to involve one and only one gravitational field (and that field must, of course, be a second-rank symmetric tensor $g_{\alpha\beta}$ which reduces to $\eta_{\alpha\beta}$ far from all gravitating bodies). If $\mathfrak{L}_{NG}$ were to involve, in addition, some other gravitational field φ, then to satisfy WEP $g_{\alpha\beta}$ and φ would have to conspire to produce *identically* the same gravitational accelerations on a test body made largely of rest mass, as on a body made largely of electromagnetic energy, as on a body made largely of internal kinetic energy, as on a body made largely of nuclear binding energy, as on a body made largely of .... This seems implausible, unless $g_{\alpha\beta}$ and φ appear everywhere in $\mathfrak{L}_{NG}$ in the same "mutually coupled" form $f(\varphi)g_{\alpha\beta}$ – in which case one can absorb $f(\varphi)$ into $g_{\alpha\beta}$ and end up with just one gravitational field in $\mathfrak{L}_{NG}$. Thus, it seems likely that WEP forces $\mathfrak{L}_{NG}$ to involve only $g_{\alpha\beta}$. This means that the theory is universally coupled – and, hence, by the theorem of Sec. V, it is a metric theory.

This argument convinces us that Schiff's conjecture is probably correct, when one restricts attention to Lagrangian-based, relativistic theories. And it is hard to see how the conjecture could fail in other types of theories.

A formal proof of Schiff's conjecture for a more limited class of theories will be given in a subsequent paper.[9]

APPENDIX: ABSOLUTE AND DYNAMICAL FIELDS IN NEWTON-CARTAN THEORY

In order to separate the absolute gravitational fields of Newton-Cartan theory from the dynamical fields, one must change mathematical representations. In place of the representation given in Table I, one can adopt the following.

1. Gravitational fields.

a. Symmetric covariant derivatives (two of them): $\underline{D}$ and $\underline{\nabla}$.

b. Scalar gravitational field: Φ.

c. Spatial metric [defined on vectors $\underline{w}$ such that $\langle \underline{\beta}, \underline{w} \rangle = 0$]: $\underline{\gamma}$.

d. Universal 1-form: $\underline{\beta}$.

(Note: t has been replaced by $\underline{\beta}$ in order to remove from the theory the "irrelevant" choice of origin of universal time; see "irrelevant variables" in Sec. II A. $\underline{D}$ and Φ will turn out to be absolute and dynamical parts of $\underline{\nabla}$; see below.)

2. Gravitational field equations.

a. $\underline{\beta}$ is perfect: $\underline{d}\,\underline{\beta} = 0$. (A1a)

b. $\underline{\beta}$ is covariantly constant: $\underline{D}\,\underline{\beta} = 0$. (A1b)

c. $\underline{D}$ is flat: $\underline{\text{Riemann}}\,(\underline{D}) = 0$. (A1c)

d. Compatibility of $\underline{D}$ and $\underline{\gamma}$:

$\underline{D}_{\underline{n}}(\underline{v}\cdot\underline{w}) = (\underline{D}_{\underline{n}}\,\underline{v})\cdot\underline{w} + \underline{v}\cdot(\underline{D}_{\underline{n}}\,\underline{w})$ for any vector $\underline{n}$, and for any spatial vectors $\underline{v}$, $\underline{w}$. (A1d)

e. Decomposition of $\underline{\nabla}$:

$\underline{\nabla} = \underline{D} + \underline{A}\otimes\underline{\beta}\otimes\underline{\beta}$, where $\underline{A}$ is the spatial vector "dual" to $\underline{d}\Phi$: $\langle \underline{d}\Phi, \underline{w} \rangle = \underline{A}\cdot\underline{w}$ for all spatial $\underline{w}$. (A1e)

f. Field equation for Φ:

$$\underline{D}\cdot\underline{A} \equiv (\text{divergence of } \underline{A}) = 4\pi\rho. \qquad \text{(A1f)}$$

3. Influence of gravity on matter. Same as in part 4 of Table I where t is *any* scalar field such that $\underline{\beta} = \underline{dt}$.

To prove that this and the formalism given in Table I are different mathematical representations of the same theory, we can show that they become identical in Galilean coordinate frames. The reduction of the formalism of Table I to a Galilean frame is performed in Exercise 12.6 of MTW.[12] The reduction of the above formalism proceeds as follows: (i) Let t be any particular scalar field such that $\underline{\beta} = \underline{dt}$. (ii) At some particular event in spacetime pick a set of basis vectors $\{\underline{e}_\alpha\}$ such that (a) $\underline{e}_1$, $\underline{e}_2$, $\underline{e}_3$ are spatial, $\langle \underline{\beta}, \underline{e}_j\rangle = 0$, and orthonormal, $\underline{e}_j \cdot \underline{e}_k = \delta_{jk}$; (b) $\underline{e}_0$ is not spatial, $\langle \underline{\beta}, \underline{e}_0\rangle \neq 0$. (iii) From each vector $\underline{e}_\alpha$ construct a vector field on all of spacetime by parallel transport with $\underline{D}$. The resulting field is unique because $\underline{D}$ is flat; and it has $\underline{D}\underline{e}_\alpha = 0$. Hence, the commutators vanish:

$$[\underline{e}_\alpha, \underline{e}_\beta] = \underline{D}_\alpha \underline{e}_\beta - \underline{D}_\beta \underline{e}_\alpha = 0.$$

This guarantees the existence of a coordinate system $\{x^\alpha\}$ in which $\underline{e}_\alpha = \partial/\partial x^\alpha$. (iv) The condition (valid in *any* coordinate frame) $\langle \underline{dx}^0, \underline{e}_j\rangle = 0$, when compared with $\langle \underline{dt}, \underline{e}_j\rangle = 0$, guarantees that the surfaces of constant x^0 and constant t are identical, i.e., $t = f(x^0)$. Moreover, because the connection coefficients of $\underline{D}$ vanish in this coordinate frame,

$$\begin{Bmatrix} \alpha \\ \beta\gamma \end{Bmatrix} \equiv \langle \underline{dx}^\alpha, \underline{D}_\gamma \underline{e}_\beta\rangle = 0, \qquad \text{(A2a)}$$

the condition $\underline{D}\underline{dt} = 0$ becomes $\partial^2 t/\partial x^\alpha \partial x^\beta = 0$; in particular, $\partial^2 t/\partial x^0 \partial x^0 = 0$, so $t = ax^0 + b$ for some constants a and b. Renormalize x^0 so $t = x^0$. (v) In the resulting coordinate frame $\underline{\beta}$, $\underline{\gamma}$, and $\underline{A}$ have components

$$\beta_0 = 1, \quad \beta_j = 0, \quad \gamma_{jk} = \delta_{jk},$$
$$A^0 = 0, \quad A^j = \partial\Phi/\partial x^j; \qquad \text{(A2b)}$$

so the field equation for Φ is Poisson's equation

$$\frac{\partial^2\Phi}{\partial x^j \partial x^j} = 4\pi\rho; \qquad \text{(A2c)}$$

and the connection coefficients of $\underline{\nabla}$ are $\Gamma^\alpha{}_{\beta\gamma} = A^\alpha t_{,\beta} t_{,\gamma}$, i.e.,

$$\Gamma^j{}_{00} = \frac{\partial\Phi}{\partial x^j}, \quad \text{all other } \Gamma^\alpha{}_{\beta\gamma} \text{ vanish}. \qquad \text{(A2d)}$$

This Galilean coordinate version of the above formalism is identical to the Galilean coordinate version of the formalism of Table I, as given in Chap. 12 of MTW.[12] Thus, the two formalisms are different mathematical representations of the same theory.

In the above formalism it is easy to verify that $\underline{D}$, $\underline{\beta}$, and $\underline{\gamma}$ are absolute gravitational fields, while Φ is a dynamical gravitational field. In fact, $\underline{D}$, $\underline{\beta}$, and $\underline{\gamma}$ are the absolute parts of $\underline{\nabla}$; Φ is its dynamical part; Eqs. (A1a)–(A1d) are the prior geometric constraints of the theory; Eq. (A1e) is the decomposition of $\underline{\nabla}$ into its absolute and dynamical parts; and Eq. (A1f) is the dynamical field equation for Φ.

The covariance group for the above mathematical representation of Newton-Cartan theory is slightly larger than that for the representation of Table I. For Table I the covariance group is MMG. For the above representation it is the direct product of MMG with a group of *internal covariance transformations*. In a Galilean frame the internal transformations are

$$\begin{Bmatrix} j \\ 0\,0 \end{Bmatrix} \to \begin{Bmatrix} j \\ 0\,0 \end{Bmatrix}' = \begin{Bmatrix} j \\ 0\,0 \end{Bmatrix} + a^j(t) = a^j(t),$$
$$\Phi \to \Phi' = \Phi - a^j(t)x^j + \text{constant}, \qquad \text{(A3)}$$
$$\text{all other variables, including } \Gamma^\alpha{}_{\beta\gamma}, \text{ left unchanged}.$$

In coordinate-free form the internal transformations are

$$\underline{D} \to \underline{D}' = \underline{D} + \underline{a}\otimes\underline{\beta}\otimes\underline{\beta}, \qquad \text{(A3'a)}$$
$$\Phi \to \Phi' = \Phi - b,$$

where $\underline{a}$ is any vector field which is covariantly constant in the surfaces of $\underline{\beta}$,

$$\underline{D}_{\underline{w}}\underline{a} = \underline{\nabla}_{\underline{w}}\underline{a} = 0 \quad \text{for all spatial vectors } \underline{w}; \qquad \text{(A3'b)}$$

and where b is any scalar field such that

$$\langle \underline{db}, w\rangle = \underline{a}\cdot\underline{w} \quad \text{for all spatial vectors } \underline{w}. \qquad \text{(A3'c)}$$

The complete symmetry group for the above mathematical representation of Newton-Cartan theory is best analyzed in a Galilean coordinate system. [Because the absolute objects are all tensors or affine connections, one can restrict attention to a single coordinate system; see Eq. (13) and associated discussion in the text.] The symmetry transformations are those which leave

$$\bar{\delta}\gamma_{jk} = \bar{\delta}\beta_\alpha = \bar{\delta}\begin{Bmatrix} \alpha \\ \beta\gamma \end{Bmatrix} = 0. \qquad \text{(A4)}$$

Clearly, the symmetry transformations include (i) spacetime translations

$$x^\alpha \to x^{\alpha\prime} = x^\alpha + c^\alpha, \tag{A5a}$$

where c^α are constants, and (ii) spatial rotations

$$x^j \to x^{j\prime} = R^{jk} x^k, \tag{A5b}$$

$\|R^{jk}\|$ a constant rotation matrix. They also include (iii) the combination of an arbitrary time-dependent spatial translation with a carefully matched internal covariance transformation

$$\left.\begin{aligned} x^j &\to x^{j\prime} = x^j + c^j(t), \quad \text{where } c^j \text{ are arbitrary functions of } t, \\ \begin{Bmatrix} j \\ 0\,0 \end{Bmatrix} &\to \begin{Bmatrix} j \\ 0\,0 \end{Bmatrix}^\dagger = \begin{Bmatrix} j \\ 0\,0 \end{Bmatrix} + \ddot{c}^j(t) \\ \Phi &\to \Phi^\dagger = \Phi - \ddot{c}^j(t) x^j \end{aligned}\right\} \quad \text{where } \ddot{c}^j = \frac{d^2 c^j}{dt^2}. \tag{A5c}$$

Note that these symmetry transformations are precisely the transformations that lead from one Galilean coordinate system to another (cf. Sec. 12.3 of MTW[12]).

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[5]Our form of Schiff's conjecture is a classical analog of Schiff's original quantum mechanical conjecture. Schiff briefly outlined his version of the conjecture on page 343 of his article in Am. J. Phys. 28, 340 (1960). So far as we know, he never pursued it in any detail until November 1970, when his interest in the issue was rekindled by a vigorous **argument** with one of us (KST) at the Caltech-JPL Conference on Experimental Tests of Gravitation Theories. Unfortunately, his sudden death 2 months later took him from us before he had a chance to bring his analysis of the conjecture to fruition.

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[19]This definition of "faithful realization" differs from that given on page 26 of JLA; we think that this is what JLA intended to say and should have said.

[20]E. Kretschmann, Ann. Phys. (Leipzig) 53, 575 (1917).

[21]For the topological properties of groups see, e.g., L. S. Pontrjagin, *Topological Groups* (Oxford Univ. Press, Oxford, 1946).

[22]This concept is due to C. W. Misner; see Chap. 17 of MTW (Ref. 12).

[23]*Note added in proof.* A. Trautman (private communication) argues that we should define as "gravitational" those phenomena which are absolute or "go away" as the amounts of mass-energy *and spin* in the experimental laboratory decrease. Such a definition, he argues, would be "in accordance with our knowledge, based on special relativity, where mass and spin are equally fundamental." He goes on to say, "I feel that it is a little misleading to arrange the definitions (by omitting the phrase 'and spin' above) so as to exclude the Einstein-Cartan theory from the framework of relativistic theories of gravity (Sec. IV of this paper). Clearly, the Einstein-Cartan theory reduces to special relativity when $G \to 0$. Moreover, in the real world, spin, charge, baryon number, etc. are always accompanied by energy and momentum. Therefore, if the amount of mass-energy in the laboratory decreases, then not only gravitational phenomena go away: so do electromagnetic and nuclear effects as well as the (so far hypothetical) effects of torsion."

We (the authors) find Trautman's argument cogent. Nevertheless, we adhere to our set of definitions — for a very pragmatic reason. By refusing to allow torsion into "relativistic theories of gravity" we make it possible for ourselves to prove certain theorems about relativistic theories which otherwise we would not know how to prove and which might well be false. An example is the theorem

on universal coupling proved in Sec. V of this paper.

For details and references on Cartan's theory with torsion, see A. Trautman, Bull. Acad. Pol. Sci. Ser. Sci. Math. Astron. Phys. 20, 185 (1972), and references cited therein.

[24]R. H. Dicke, lectures in *Relativity, Groups, and Topology*, edited by C. DeWitt and B. DeWitt (Gordon and Breach, New York, 1964).

[25]For a more detailed discussion of WEP see Chap. 38 of MTW—where WEP goes under the name "universality of free fall".

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[27]See, e.g., Chap. 10 of MTW.

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[29]This definition of "metric theory" originated in Chap. 39 of MTW. Here and henceforth we shall adhere to it, even though earlier work by our group (e.g., Ref. 1) used a slightly less restrictive definition. (Any theory that is "metric" according to the present definition is also "metric" according to the old definition.)

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[32]C. M. Will and K. Nordtvedt, Jr., Astrophys. J. 177, 757 (1972); also earlier references by Nordtvedt and by Will cited therein. For reviews see Refs. 2, 3, 4, and 12.

[33]We have adapted the principle of universal coupling from R. V. Wagoner, Phys. Rev. D 1, 3209 (1970). Wagoner enunciated this principle only for the special case of scalar-tensor theories, and he gave it the more restrictive name "principle of mutual coupling." However, our concept is a straightforward generalization of his.

[34]To see more clearly why no gravitational fields other than $g_{\alpha\beta}$ can enter the Euler-Lagrange equations, argue as follows: The only gravitational effects which vanish as the size of the frame vanishes arise from terms of a Taylor series type expansion of some gravitational field(s) B. But if there are any B other than $g_{\alpha\beta}$, then somewhere in spacetime there will be local Lorentz frames of $g_{\alpha\beta}$ in which the lowest order Taylor series term of some of the B does not vanish, thus violating the local validity of special relativity.

[35]This is a classical version of Schiff's original quantum-mechanical line of reasoning (Ref. 5).

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Analysis of the Belinfante-Swihart Theory of Gravity*

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We show that the Belinfante-Swihart (BS) theory can be reformulated in a representation in which uncharged matter responds to gravity in the same way as in metric theories. The BS gravitationally modified Maxwell equations can also be put into metric form to first order in the deviations of the physical metric from flat space, but not to second order; consequently, the theory is nonmetric except in first order. We also show that the theory violates the high-precision Eötvös-Dicke experiment, but cannot be ruled out by the gravitational precession of gyroscopes.

I. INTRODUCTION AND SUMMARY

This paper analyzes the most complete and extensively developed nonmetric theory that exists: the 1957 theory of Belinfante and Swihart.[1-3] Belinfante and Swihart (BS) constructed their theory as a Lorentz-symmetric[4] linear field theory which would be easily quantized. However, as we shall show, in terms of measurable quantities the theory has all the nonlinearities of typical "curved-spacetime" theories. Moreover, it is nearly a metric[4] theory: We construct a new mathematical representation which has metric form to first order in deviations of the physical metric from flatness, but does not have metric form to higher orders.

Section II gives a brief summary of the original BS representation. Included are discussions of nonlinearities and the behavior of rods and clocks. Section III presents our new mathematical representation of the theory. Section IV gives a prescription for obtaining the post-Newtonian limit[5,6] of the theory, and Sec. V considers various experimental tests. Contrary to previous calculations[7] it is found that both the geodetic and the Lens-Thirring precessions of gyroscopes[8] cannot dis-