Bulletin of the Seismological Society of America, Vol. 109, No. 6, pp. 2738–2740, December 2019, doi: 10.1785/0120190220

Short Note

On the Integrated Surface Uplift for Dip-Slip Faults

by Paul Segall and Elías Rafn Heimisson*

Abstract Interferometric Synthetic Aperture Radar observations often provide maps of vertical displacement that can be integrated to estimate an uplift volume. Relating this measure to source processes requires a model of the deformation. Bignami *et al.* (2019) argue that the negative uplift volume associated with the 2016 Amatrice–Norcia, central Italy, earthquake sequence requires a coseismic volume collapse of the hanging wall. Using results for dip-slip dislocations in an elastic half-space we show that $V_{\text{uplift}} = (P/4)(1 - 2\nu)\sin(2\delta)$, in which P is the seismic potency, ν is the Poisson's ratio, and δ is the fault dip, consistent with an earlier result of Ward (1986). For reasonable estimates of net potency for the 2016 Amatrice–Norcia sequence, this simple formula yields uplift volume estimates close to that observed. We conclude that the data are completely consistent with elastic dislocation theory and do not require a volume collapse at depth.

Introduction

The availability of data from Interferometric Synthetic Aperture Radar (InSAR) satellites provides maps of surface displacement in a wide range of tectonic and volcanic environments. With two or more imaging geometries, for example, from ascending and descending satellite passes, it is possible to reconstruct the vertical displacement (Wright *et al.*, 2004). This provides the opportunity to integrate the vertical displacement field to estimate an effective uplift volume. Relating this quantity to source processes requires a model of the deformation, which can sometimes yield unintuitive results. For example, a point source of volume expansion (also known as Mogi source) generates an uplift volume that, except in the case of an incompressible half-space, exceeds the source volume change (e.g., Segall, 2010, chapter 7).

Bignami *et al.* (2019) analyzed InSAR data associated with the 2016 Amatrice–Norcia, central Italy, earthquake sequence. Vertical displacements between 24 August 2016 and November 2016 exhibit uplift of as much as 14 cm, and subsidence of up to 100 cm. They computed the integrated subsidence of the hanging wall of 0.12 km^3, whereas the integrated uplift of the footwall is only 0.02 km^3. The authors argue that this "unbalance" of 0.10 km^3 is inconsistent with elastic rebound and requires collapse of a previously dilated zone within the hanging wall to "accommodate the hanging wall settlement". However, for a compressible earth we should not expect the integrated surface displacements to vanish. Indeed, Ward (1986) noted that normal faults lead to surface volume loss. Here, we derive an expression for the surface volume change due to dip-slip faulting employing the well-known results of Okada (1985). We then ask if the InSAR observations from Amatrice–Norcia are consistent with conventional dislocation theory, or if they require some form of dilatant collapse.

Results

Below we show that for a point dip-slip source in a homogeneous, isotropic elastic half-space that the integral of the vertical displacements on the free surface is given by

$$V_{\text{uplift}} = \frac{P}{4}(1 - 2\nu)\sin(2\delta), \tag{1}$$

in which P is the seismic potency, ν is the Poisson's ratio, and δ is the fault dip, defined as in Okada (1985). V_{uplift} vanishes for an incompressible material ($\nu = 0.5$), or for vertical and horizontal faults, as it must by symmetry. However, for $\nu = 0.25$, $\delta = 45°$ we have $V_{\text{uplift}} = P/8$. Further, the result is independent of source depth. Because any dislocation can be represented as a summation of point sources, equation (1) also applies to a finite dislocation, or to slip on multiple faults. Equation (1) is consistent with the result of Ward (1986), which gives V_{uplift} for a general moment tensor.

Derivation

For a point dip-slip dislocation with potency $P = s\Sigma$ in which s is the fault slip and Σ is the fault area, the vertical

*Now at Caltech Seismological Laboratory, South Mudd 270, 1200 East California Boulevard, Pasadena, California 91125 U.S.A.; eheimiss@caltech.edu.

displacement u_3 evaluated at the surface $(x_3 = 0)$ of an elastic half-space is

$$u_3(x_1, x_2, x_3 = 0) = -\frac{P}{2\pi}\left[\frac{3dpq}{r^5} - I_5^0 \sin\delta\cos\delta\right] \quad (2)$$

(Okada, 1985), in which d is the depth of the fault and δ is the dip; if $\sin(2\delta) > 0$ the slip is reverse. Also, following Okada (1985) notation

$$p = x_2\cos\delta + d\sin\delta, \quad (3a)$$

$$q = x_2\sin\delta - d\cos\delta, \quad (3b)$$

$$r^2 = x_1^2 + x_2^2 + d^2 = x_1^2 + p^2 + q^2, \quad (3c)$$

$$I_5^0 = (1-2\nu)\left[\frac{1}{r(r+d)} - x_1^2\frac{2r+d}{r^3(r+d)^2}\right]. \quad (3d)$$

Here, x_1 is the coordinate parallel to strike and x_2 is perpendicular.

The volume of the uplift is the integral over the surface of the vertical displacement u_3,

$$V_{\text{uplift}} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_3(x_1, x_2, x_3 = 0)dx_1dx_2. \quad (4)$$

Consider the first term in equation (2), independent of ν. The integral is proportional to

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\frac{(x_2^2-d^2)\sin\delta\cos\delta}{r^5} + \frac{x_2d(\sin^2\delta-\cos^2\delta)}{r^5}\right]dx_1dx_2. \quad (5)$$

The second term in equation (5) is odd in x_2 and therefore vanishes when integrated over the full domain. For the first term in equation (5), define $\rho^2 = x_1^2 + x_2^2$ and then transform the integral to polar coordinates (ρ, θ), noting $x_2 = \rho\sin(\theta)$ and $r^2 = \rho^2 + d^2$. The area element on the free surface becomes $dx_1dx_2 \to \rho d\rho d\theta$.

The first term in (5) thus becomes

$$\sin\delta\cos\delta\int_0^{2\pi}\int_0^{\infty}\frac{\rho^2\sin^2(\theta) - d^2}{(\rho^2+d^2)^{5/2}}\rho d\rho d\theta = \quad (6a)$$

$$\pi\sin\delta\cos\delta\int_0^{\infty}\frac{\rho^2 - 2d^2}{(\rho^2+d^2)^{5/2}}\rho d\rho = \quad (6b)$$

$$\pi\sin\delta\cos\delta\int_d^{\infty}\frac{r^2-3d^2}{r^4}dr = 0. \quad (6c)$$

Here, the first step makes use of $\sin^2(\theta) = (1-\cos(2\theta))/2$, and the second that $\rho d\rho = rdr$. Thus, the first integral vanishes, as it must because it is independent of ν and V_{uplift} must vanish for an incompressible material.

The integral over the first term in I_5^0 is proportional to

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{r(r+d)}dx_1dx_2 \to \int_0^{2\pi}\int_d^{\infty}\frac{drd\theta}{(r+d)} = 2\pi\int_d^{\infty}\frac{dr}{(r+d)}. \quad (7)$$

Following the same procedure as aforementioned, the second term becomes

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x_1^2\frac{2r+d}{r^3(r+d)^2}dx_1dx_2 \to -\pi\int_d^{\infty}\frac{(r^2-d^2)(2r+d)dr}{r^2(r+d)^2}. \quad (8)$$

Combining equations (7) and (8), and simplifying yields

$$\pi\int_d^{\infty}\left[\frac{2}{(r+d)} - \frac{(r-d)(2r+d)}{r^2(r+d)}\right]dr = \quad (9a)$$

$$\pi\int_d^{\infty}\frac{d\ dr}{r^2} = \pi. \quad (9b)$$

Combining this with equations (4) and (2) yields equation (1).

Discussion and Conclusion

We have shown that for a compressible earth the uplift volume is generally positive for reverse faults $(\sin(2\delta) > 0)$ and negative for normal faults $(\sin(2\delta) < 0)$. The vertical displacements are antisymmetric for vertical faults, so the net volume change is zero.

The Amatrice–Norcia sequence has a net moment magnitude of M_w 6.5, corresponding to a seismic moment of 6.2×10^{18} N · m. For a shear modulus 10^{10} Pa, we estimate P of 6.2×10^8 m^3. For a dip $\delta = -50°$ and $\nu = 0.25$, equation (1) yields a net volume decrease of 0.08 km^3, which is within 20% of the Bignami *et al.* (2019) estimate of 0.10 km^3.

We must be somewhat cautious of the simple estimate, which does not account for heterogeneous elastic properties that are certain to exist. There is also uncertainty in the proper value of shear modulus to use in deriving the potency, as well as the average fault dip. We do not discriminate, in this simple estimate, between slip on the main and antithetic fault. Furthermore, the analytical result integrates over the entire free surface, whereas the Bignami *et al.* (2019) calculation was over a finite domain in which the absolute value of the vertical displacement exceeded 3 cm. We have found that, because the displacements decay slowly, limiting the integration domain can substantially overpredict the computed volume change. For a normal fault, this tends to underestimate the smaller uplift volume of the footwall. For example, only integrating over the domain that is ±3 times the source depth yields a volume estimate that is a factor of two larger (in absolute value) than that given by equation (1). This effect could alone explain the 20% discrepancy between our back of the envelope calculation and the Bignami *et al.* (2019) result.

We conclude that a 20% discrepancy is insignificant, and the InSAR observations from the Amatrice–Norcia sequence are consistent with conventional dislocation theory. We do not claim that the elastic dislocation model is unique. Occam's razor, however, suggests that a simpler, well-tested theory (elastic dislocation theory) should be preferred.

Data and Resources

This is a theoretical article and contains no data.

Acknowledgments

The authors thank Bill Ellsworth for bringing this problem to our attention. The authors thank Richard Walters, Carlo Doglioni, and an anonymous reviewer for their comments.

References

Bignami, C., E. Valerio, E. Carminati, C. Doglioni, P. Tizzani, and R. Lanari (2019). Volume unbalance on the 2016 Amatrice-Norcia (Central Italy) seismic sequence and insights on normal fault earthquake mechanism, *Sci. Rep.* **9,** no. 1, 4250.

Okada, Y. (1985). Surface deformation due to shear and tensile faults in a half-space, *Bull. Seismol. Soc. Am.* **75,** no. 4, 1135–1154.

Segall, P. (2010). *Earthquake and Volcano Deformation*, Princeton University Press, Princeton, New Jersey, 432 pp.

Ward, S. N. (1986). A note on the surface volume change of shallow earthquakes, *Geophys. J. Int.* **85,** no. 2, 461–466.

Wright, T. J., B. E. Parsons, and Z. Lu (2004). Toward mapping surface deformation in three dimensions using InSAR, *Geophys. Res. Lett.* **31,** no. 1, doi: 10.1029/2003GL018827.

Geophysics Department
Stanford University
397 Panama Mall
Stanford, California 94305 U.S.A.
segall@stanford.edu

Manuscript received 29 August 2019;
Published Online 12 November 2019