Buffer Occupancy of Statistical Multiplexers with Periodic Interchangeable Traffic in ATM Networks

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Abstract — **In this paper we analyze the buffer occupancy in a statistical multiplexer in ATM networks for a special type of traffic, namely, *periodic interchangeable* (PI) traffic. Certain generalized *Ballot theorem* is applied to analyze the problem. Explicit formulas for the expected buffer occupancy are derived.**

I. Introduction

An important concept in ATM networks is the efficient sharing of link capacities through statistical multiplexing of traffic sources. Buffering is provided at a statistical multiplexer to manage traffic fluctuations when the instantaneous rate of the aggregate incoming traffic exceeds the capacity of the outgoing link. The analysis of buffer occupancy for a statistical multiplexer with burst-constrained traffic is important to maintain quality-of-service guarantees in ATM networks. In this paper, we analyze the buffer occupancy for a statistical multiplexer with a special type of burst-constrained traffic, *periodic interchangeable* traffic.

II. Ballot Theorem and Interchangeable Random Variables

Ballot Theorem [2]. If in a ballot candidate A scores a votes and candidate B scores b votes and if $a \geq b$, then the probability that throughout the counting the number of votes registered for A is always greater than the number of votes registered for B is given by

$$P(a,b) = \frac{a-b}{a+b}, \tag{1}$$

provided that all possible voting records are equally likely.

Definition. Random variables $(X_1, X_2, \cdots, X_M)$ are *cyclically interchangeable* if $(X_1, X_2, \cdots, X_M)$ take the value $(x_1, x_2, \cdots, x_M)$, then they have the same probability for taking values of every cyclic permutation of $(x_1, x_2, \cdots, x_M)$; they are called *interchangeable* if they have the same probability for taking values of every permutation of $(x_1, x_2, \cdots, x_M)$.

Theorem (Takacs) [2]. If $(A_1, A_2, \cdots, A_M)$ are cyclically interchangeable random variables such as $\sum_{i=1}^{M} A_i = N$, then

$$P\{\sum_{j=1}^{i} A_j < i, \forall i \in [1, M]\} = \begin{cases} 1 - \frac{N}{M} & \text{if } N < M \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

[1]This work was supported by a grant from PacBell

III. Statistical Multiplexers with Periodic Interchangeable Traffic

Consider a statistical multiplexer with an infinite buffer. Assume time is slotted and it takes a slot of time for the transmission line to transmit a cell. Suppose source i, $1 \leq i \leq K$, generates n_i cells within a period of M slots; each source produces cells periodically with the same period M. The utilization factor $\rho = \frac{N}{M} < 1$. Define

$$\begin{cases} Q_k &= \text{buffer occupancy at the end of the } k\text{th slot} \\ A_k &= \text{number of arriving cells in the } k\text{th slot,} \end{cases}$$

and $\bar{Q}$ the expected buffer occupancy. Note that $\sum_{i=1}^{M} A_i = \sum_{i=1}^{K} n_i = N$. Assume random variables $A_1, A_2, \cdots, A_M$ are interchangeable. Then, the traffic model is called *periodic interchangeable* (PI) traffic, which is a (σ, ρ) regulated traffic source with $\sigma = \rho(M-1)$. A PI traffic in which each source generates cell uniformly within a period is called *uniform* PI (UPI) traffic. The UPI traffic in which each source generates only one cell within a period is called the *unit* UPI (UUPI) traffic [1]. Applying Takacs' generalized Ballot theorem, we can prove the following theorems:

Theorem 1. For a statistical multiplexer with UPI traffic,

$$\bar{Q} = \sum_{q=1}^{N} \sum_{r=1}^{N-q} \frac{M-N+q}{M-r} \sum_{\substack{\sum_{i=1}^{K} m_i = q+r \\ 0 \leq m_i \leq n_i}} \prod_{i=1}^{K} \frac{\binom{r}{m_i}\binom{M-r}{n_i-m_i}}{\binom{M}{n_i}}. \tag{3}$$

Theorem 2. For a statistical multiplexer with UUPI traffic, when K is sufficiently large,

$$\bar{Q} \begin{cases} = \frac{1}{2}(\sqrt{\frac{\pi K}{2}} - \frac{1}{3}) + o(1) & \rho = 1 \\ \leq \frac{\rho\alpha(1-\rho)}{1-\alpha} + o(1) & \rho < 1. \end{cases} \tag{4}$$

IV. Conclusion

In this paper we applied Takacs' generalized ballot theorem to analyze the queue occupancy in a statistical multiplexer in ATM networks for a special type of burst-constrained traffic, *periodic interchangeable* (PI) traffic. Explicit formulas for the expected queue occupancy are derived for special PI traffic: UPI and UUPI. The result can shed light on the study of the worst case performance of a statistical multiplexer with burst-constrained traffic in ATM networks.

References

[1] P. Humblet, A. Bhargava, and M. G. Hluchyj, "Ballot theorems applied to the transient analysis of nD/D/1 queues", IEEE/ACM Trans. on Networking, vol. 1, no. 1, Feb. 1993.

[2] L. Tackas, *Combinatorial Methods in the Theory of Stochastic Processes*, New York: Wiley, 1967.