## Supplementary information

## A super-massive Neptune-sized planet

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## Supplementary Informations

## Description of the dynamical simulations from Sect. 2.12.

The first simulation considers 2 planets of 40 Earth masses around a solar-mass star. Their initial orbital elements are:
$a_{1}=0.02000 \mathrm{au}, \mathrm{e}_{1}=0.006, \mathrm{i}_{1}=1.27 \mathrm{deg} ., \Omega_{1}=54.8 \mathrm{deg} ., \omega_{1}=297.5 \mathrm{deg}, M_{1}=76.3 \mathrm{deg}$.
$a_{2}=0.02121$ au, $e_{2}=0.002, i_{2}=1.02$ deg., $\Omega_{2}=112.6$ deg., $\omega_{2}=227.2 \mathrm{deg}, M_{2}=110.9 \mathrm{deg}$.
The two planets merged into a single one with orbital elements:
$a=0.02065 \mathrm{au}, \mathrm{e}=0.007, \mathrm{i}_{2}=0.76 \mathrm{deg}$.
The second simulation considers 4 planets of 20 Earth masses. Their initial orbital elements are:
$a_{1}=0.02000 \mathrm{au}, e_{1}=0.006, i_{1}=1.27$ deg., $\Omega_{1}=54.8$ deg., $\omega_{1}=297.5 \mathrm{deg}, M_{1}=76.3 \mathrm{deg}$.
$a_{2}=0.02096 \mathrm{au}, \mathrm{e}_{2}=0.002, \mathrm{i}_{2}=1.02$ deg., $\Omega_{2}=112.6$ deg., $\omega_{2}=227.2 \mathrm{deg}, \mathrm{M}_{2}=110.9 \mathrm{deg}$.
$a_{3}=0.02185 \mathrm{au}, \mathrm{e}_{3}=0.044, \mathrm{i}_{3}=0.31$ deg., $\Omega_{3}=4.9$ deg., $\omega_{3}=18.3 \mathrm{deg}, M_{3}=307.8 \mathrm{deg}$.
$a_{4}=0.02258 \mathrm{au}, \mathrm{e}_{4}=0.004, \mathrm{i}_{4}=1.07$ deg., $\Omega_{4}=51.5$ deg., $\omega_{4}=322.0 \mathrm{deg}, \mathrm{M}_{4}=253.0 \mathrm{deg}$.
The planets merged into a single one with orbital elements:
$a=0.02121 \mathrm{au}, \mathrm{e}=0.012, \mathrm{i}_{2}=0.81 \mathrm{deg}$.
In the third simulation we considered 8 planets of 10 Earth masses. Their initial orbital elements are:
$a_{1}=0.02000 \mathrm{au}, e_{1}=0.006, i_{1}=1.27$ deg., $\Omega_{1}=54.8$ deg., $\omega_{1}=297.5 \mathrm{deg}, M_{1}=76.3 \mathrm{deg}$.
$\mathrm{a}_{2}=0.02076 \mathrm{au}, \mathrm{e}_{2}=0.002, \mathrm{i}_{2}=1.02 \mathrm{deg} ., \Omega_{2}=112.6$ deg., $\omega_{2}=227.2 \mathrm{deg}, \mathrm{M}_{2}=110.9 \mathrm{deg}$.
$a_{3}=0.02146$ au, $e_{3}=0.044, i_{3}=0.31$ deg., $\Omega_{3}=4.9$ deg., $\omega_{3}=18.3$ deg, $M_{3}=307.8$ deg.
$a_{4}=0.02203 \mathrm{au}, \mathrm{e}_{4}=0.004, \mathrm{i}_{4}=1.07$ deg., $\Omega_{4}=51.5 \mathrm{deg} ., \omega_{4}=322.0 \mathrm{deg}, \mathrm{M}_{4}=253.0 \mathrm{deg}$.
$a_{5}=0.02283 \mathrm{au}, \mathrm{e}_{5}=0.037, \mathrm{i}_{5}=0.13 \mathrm{deg} ., \Omega_{5}=247.6 \mathrm{deg} ., \omega_{5}=271.0 \mathrm{deg}, \mathrm{M}_{5}=216.5 \mathrm{deg}$.
$a_{6}=0.02344 \mathrm{au}, e_{6}=0.042, i_{6}=0.84$ deg., $\Omega_{6}=245.6$ deg., $\omega_{6}=121.8$ deg, $M_{6}=203.5$ deg.
$a_{7}=0.02431 \mathrm{au}, \mathrm{e}_{7}=0.044, \mathrm{i}_{7}=0.67 \mathrm{deg} ., \Omega_{7}=183.5 \mathrm{deg} ., \omega_{7}=99.4 \mathrm{deg}, \mathrm{M}_{7}=259.5 \mathrm{deg}$.
$\mathrm{a}_{8}=0.02521 \mathrm{au}, \mathrm{e}_{8}=0.031$, $\mathrm{i}_{8}=0.28$ deg., $\Omega_{8}=9.3$ deg., $\omega_{8}=268.3 \mathrm{deg}, \mathrm{M}_{8}=37.9 \mathrm{deg}$.
This system results into two planets of 50 and 30 Earth masses with, respectively, orbital elements:
$a_{1}=0.02026 \mathrm{au}, e_{1}=0.043, i_{1}=0.46 \mathrm{deg}$.
$a_{2}=0.02642$ au, $e_{2}=0.081, i_{2}=0.63 \mathrm{deg}$.
The second simulation depicts a Jupiter-mass planet on an orbit with a $1 \mathrm{au}, \mathrm{e}=0.98$ (so that its perihelion distance is 0.02 au ) and $\mathrm{i}=10$ deg.

We simulated the effect of this planet on a disk of 1000 test particles (representing planetesimals or planets of smaller mass with respect ot the giant planet) with $0.02 \mathrm{au}<\mathrm{a}<0.06 \mathrm{au}, \mathrm{e}<0.1$ and $\mathrm{i}<2.5$
degrees. We simulated the system for 150'000 years, at the end of which only 2 particles survive. 309 (31\%) have been accreted by the giant planet.

We then repeated the simulation with a disk of 1000 test particles with $0.1 \mathrm{au}<\mathrm{a}<0.3 \mathrm{au}$ and then with $0.5 \mathrm{au}<\mathrm{a}<1.5 \mathrm{au}$, both still with $\mathrm{e}<0.1$ and $\mathrm{i}<2.5$ deg.. Only 65 and 20 particles from disk disks have been accreted by the giant planet, namely 6.5 and $2 \%$ of the total. This shows that the efficiency of accretion of particles by the eccentric giant planets falls rapidly with the distance of the particles from the star.

All these simulations have been performed using the simplectic swift_symba5 integrator (properly referenced in the main text), with a timestep of $5 \times 10^{-5}$ years.

Observed-Calculated ( $\mathrm{O}-\mathrm{C}$ ) time of transits figure.


The « observed» transit times of TOI-1853b were evaluated by fitting its light curves with the orbital period fixed at its best-fitting value and leaving each time of transit as free to vary. Then the observed times were compared with the «calculated» (expected) times from the orbital period. We found no evidence of Transit Time Variations (TTV) as all the observed transit times are compatible with their expected value within $\approx 1$-sigma. The periodogram of $\mathrm{O}-\mathrm{C}$ does not result in any relevant peaks either.

Indeed, considering the mass and orbital period of TOI-1853b, the shallow transit depth, and also the lack of clear massive planets nearby (see Sect. 2.9), we didn't expect to detect TTVs unless the planet was being rapidly engulfed by the host star (which is unlikely according to Sect. 2.10).

