

# Cosine-Modulated FIR Filter Banks Satisfying Perfect Reconstruction

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**Abstract**—It is well known that FIR filter banks, satisfying the perfect reconstruction (PR) property, can be obtained by cosine modulation of a linear-phase prototype of length  $N = 2M$  ( $M$  is the number of channels) when certain constraints are imposed on the prototype. Recently, this result was extended for the case when  $N = 2mM$  ( $m$  is an arbitrary positive integer). In this paper, we obtain a necessary and sufficient condition on the  $2M$  polyphase components of a linear-phase prototype filter of length  $N = 2mM$ , such that the polyphase component matrix of the modulated filter bank is lossless. The losslessness of the polyphase component matrix, in turn, is sufficient to ensure that the analysis/synthesis system satisfies PR. Using this result, a new design procedure is presented (based on the two-channel lossless lattice). This enables the design of a large class of FIR-PR filter banks (and includes the  $N = 2M$  case). It is shown that this approach requires fewer parameters to be optimized than in the pseudo-QMF designs and in the lossless lattice based PR-QMF designs (for equal length filters in the three designs). This advantage becomes significant when designing long filters for large  $M$ . The design procedure and its other advantages are described in detail. Design examples and comparisons are included.

## I. INTRODUCTION

IN multirate digital signal processing (DSP), the topic of quadrature mirror filters (QMF's) has received widespread attention [1]–[21]. An  $M$ -channel maximally decimated QMF bank is shown in Fig. 1(a). QMF banks are used in a wide range of speech, image, and other applications [1], [2], which involve the splitting of an input signal  $x(n)$  into subbands and, finally, the reconstruction of the original signal. The two familiar approaches to  $M$ -channel QMF design are the perfect reconstruction QMF banks [4]–[15] and the pseudo-QMF banks [16]–[21]. The tradeoffs between the two are that the latter has an efficient design procedure (only the prototype filter is designed) while the former achieves perfect reconstruction (PR) of the input (i.e., without aliasing, magnitude, or phase distortions). Owing to their attractive features, PR-QMF banks are of particular interest. However, the conventional approaches for PR-QMF design [4], [5], [7],

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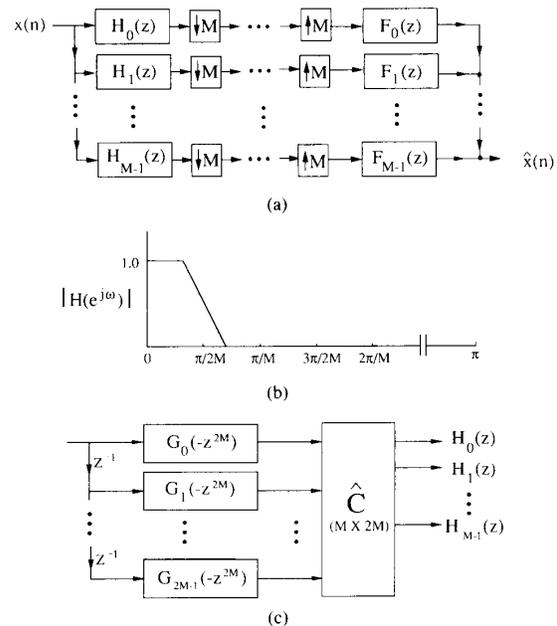


Fig. 1. (a) The  $M$ -channel maximally decimated QMF circuit. (b) The desired response of the prototype  $H(z)$ . (c) Polyphase implementation of the cosine-modulated analysis filter bank.

[8], [15] require the optimization of a nonlinear objective function of a large number of parameters (particularly for large values of  $M$ ). In this paper, we present a method, which, while retaining all the attractive features of modulated filter banks (e.g., only the prototype filter is designed), also satisfies the PR property. The analysis and synthesis filters are of equal length (an arbitrary multiple of  $2M$ ).

It is a known result [10]–[12] that FIR filter banks satisfying PR can be obtained by the modulation of a linear-phase prototype of length  $N = 2M$ , when certain constraints are imposed on the prototype. Recently, in [13], this result was extended for the case  $N = 2mM$  ( $m$  is an arbitrary positive integer) and a necessary and sufficient condition for PR was obtained (in the time domain). We understand that a manuscript [28], describing the full details, is currently under review.

In this paper, we derive an equivalent necessary and sufficient condition involving the  $2M$  polyphase components of a linear-phase FIR filter  $H(z)$  (length  $N = 2mM$ ).

This approach throws additional light on the problem and places in evidence its relation to lossless QMF banks [4]. It also yields an efficient design method where pairs of polyphase components are designed using the two-channel lossless lattice structure [22]. This enables the design of a large class of FIR-PR modulated filter banks (with  $N = 2mM$ ).

The main advantages of the proposed method are summarized below.

1) These filter banks satisfy the perfect reconstruction (PR) conditions.

2) The technique can be used to design filter banks for an arbitrary number of channels ( $M$ ).

3) The analysis and synthesis filters are of equal length ( $N$ ).

4) The analysis and synthesis filters are obtained by cosine modulation of the prototype filter. Hence they can be implemented very efficiently. Owing to the modulation that is used, the analysis and synthesis filters do not have linear phase even though the prototype has linear phase.

5) This method requires half as many parameters to be optimized compared to the pseudo-QMF method. This is also many fewer parameters than optimized in the lossless lattice based PR-QMF designs.

6) The objective function used in the optimization is very simple and it involves only the stopband energy of the prototype. In comparison, the objective function in pseudo-QMF designs includes the stopband energy of the prototype and a "flatness constraint," while in the lossless lattice based PR-QMF designs, it includes the stopband energies of all the  $M$  filters in the filter bank. Hence, the evaluation of the objective function (which is required at every iteration of the optimization) is simpler.

7) The new scheme is such that the  $2M$  polyphase components of the prototype  $H(z)$  can be grouped into  $M$  power complementary pairs. If each pair is implemented in a structurally power complementary manner, then the PR property is retained even in the presence of coefficient quantization. This is achieved by implementing each power complementary pair by a two-channel lossless lattice.

8) The optimization (to obtain the prototype filter) is done directly on the lattice parameters. This enables us to optimize the prototype response (while it is guaranteed that the modulated filter bank will satisfy the PR property).

It has been verified by explicit computation that good pseudo-QMF designs [16]–[21] are such that the polyphase component matrix  $E(z)$  of the analysis filter bank is "almost" lossless. This is consistent with the "almost" perfect reconstruction property of these cosine-modulated filter banks (see Section II-B). We can summarize by saying that this paper incorporates "exact" losslessness into pseudo-QMF techniques and, thereby, enables the PR property to be satisfied, while retaining all the attractive features of modulated filter banks.

*Outline:* The paper is outlined as follows. In Section II, the cosine modulation used in pseudo-QMF banks is men-

tioned. An important observation about pseudo-QMF banks is presented. The polyphase representation of the analysis/synthesis bank in terms of the modulation matrix and the polyphase components of the prototype filter is introduced. This notation is used in the subsequent sections, which deal with cosine-modulated PR filter banks. In Section III, we consider modulated filter banks which satisfy the PR property. In this section, it is shown that for  $E(z)$  (the polyphase component matrix of the analysis filter bank) to be lossless, it is necessary and sufficient that appropriate pairs of polyphase components of the prototype  $H(z)$  are power complementary. The losslessness of  $E(z)$  is sufficient to ensure that the analysis/synthesis system satisfies the perfect reconstruction (PR) property.

The design of prototype filters of modulated PR banks is considered next. Section IV-A contains a description of the two-channel lossless lattice. In Section IV-B, it is shown how the two-channel lossless lattice can be used to ensure that the prototype filter satisfies the condition for PR obtained in Section IV. Sections IV-C to -E deal with the design procedure (for the prototype filter) based on the two-channel lossless lattice. In Section IV-F, an efficient implementation of the modulated PR filter bank is presented along with a comparison of its implementation complexity with that of pseudo-QMF banks. Section V includes design examples to demonstrate the various aspects of the design procedure and a detailed comparison between modulated PR filter banks (designed by the approach proposed in this paper) and pseudo-QMF banks.

Appendix A contain the proofs of some identities (pertaining to the properties of the cosine-modulation matrix) which are essential in the derivation of the necessary and sufficient condition in Section III. In Appendix B, the properties of the two-channel modulated PR filter banks are discussed. It is shown that these filters satisfy the same condition as the ones in the the well-known two-channel PR-QMF design approach [6].

*Notation:* Boldface letters indicate vectors and matrices. Superscript  $T$  and  $\dagger$  denote transposition and transposed conjugation, respectively. The tilde accent on a function  $F(z)$  is defined such that  $\tilde{F}(z) = F_*^T(z^{-1})$ ,  $\forall z$ , where the asterisk (\*) subscript denotes the conjugation of coefficients. Further,  $J_M$  stands for the  $M \times M$  "reverse operator." For example,

$$J_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

## II. MODULATED FILTER BANKS

In this section, we briefly mention the cosine modulation used in pseudo-QMF banks. An important observation about pseudo-QMF banks is presented and this serves to motivate the work presented in this paper. The polyphase implementation of cosine-modulated filter banks and the related notation are also introduced.

### A. Pseudo-QMF Banks

Pseudo-QMF banks belong to the family of modulated filter banks. Pseudo-QMF theory is well known [16]–[21] and is widely used. In pseudo-QMF banks, the analysis filter  $H_k(z)$  and the synthesis filters  $F_k(z)$  are obtained by the cosine modulation of a linear-phase, low-pass prototype filter  $H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$ , as shown below:

$$h_k(n) = 2h(n) \cos \left( (2k+1) \frac{\pi}{2M} \left( n - \frac{N-1}{2} \right) + \theta_k \right), \quad 0 \leq n \leq N-1 \quad (1)$$

$$f_k(n) = 2h(n) \cos \left( (2k+1) \frac{\pi}{2M} \left( n - \frac{N-1}{2} \right) - \theta_k \right), \quad 0 \leq n \leq N-1 \quad (2)$$

where  $h_k(n)$  and  $f_k(n)$  are the impulse responses of  $H_k(z)$  and  $F_k(z)$ , respectively. In this paper, we will use the choice

$$\theta_k = (-1)^k \frac{\pi}{4}, \quad 0 \leq k \leq M-1 \quad (3)$$

which yields the same modulated filters as in [10], [16] (except for a possible scale factor of  $-1$  on some of the filters). From (1) and (2), we can verify that the analysis and synthesis filters are related as

$$f_k(n) = h_k(N-1-n) \quad \text{and} \quad F_k(z) = z^{-(N-1)} \tilde{H}_k(z) \quad (4)$$

for  $0 \leq k \leq M-1$ .  $T(z)$ , the overall transfer function of the analysis/synthesis system, has linear phase. However, the filters  $H_k(z)$  and  $F_k(z)$  do not have linear phase.

### B. A Key Observation About Pseudo-QMF Banks

Let  $\mathbf{h}(z) = [H_0(z) \ H_1(z) \ \cdots \ H_{M-1}(z)]^T$  be the analysis filters of a pseudo-QMF bank. Using type I polyphase decomposition [2],  $\mathbf{h}(z)$  can be expressed as

$$\mathbf{h}(z) = \mathbf{E}(z^M) \mathbf{e}_M(z) \quad (5)$$

where  $\mathbf{E}(z)$  is the polyphase component matrix of the filter bank and  $\mathbf{e}_M^T(z) = [1 \ z^{-1} \ \cdots \ z^{-(M-1)}]$ . The new observation is a property of the matrix  $\mathbf{E}(z)$  of pseudo-QMF banks. We have the following result from [4].

*Fact:* In any perfect reconstruction QMF bank, if the analysis and synthesis filters are related as

$$F_k(z) = cz^{-(N-1)} \tilde{H}_k(z), \quad 0 \leq k \leq M-1 \quad (6)$$

where  $c$  is a nonzero constant, then  $\mathbf{E}(z)$ , the polyphase component matrix of the analysis filter bank is necessarily lossless.  $\diamond$

Based on the known pseudo-QMF design techniques, it is possible to obtain designs such that the aliasing error and the reconstruction error of the analysis/synthesis can be made very small. The overall transfer function  $T(z)$  has approximately unit gain at all frequencies. Hence, it is intuitively expected (by using the Fact mentioned above)

that the matrix  $\mathbf{E}(z)$  of the pseudo-QMF bank will be ‘‘approximately’’ lossless, i.e.,  $\mathbf{E}(z)$  satisfies the condition  $\tilde{\mathbf{E}}(z)\mathbf{E}(z) \approx \mathbf{I}_M$ , with the nondiagonal terms of the LHS being small, but not necessarily zero. This result was verified using a number of design examples, one of which is shown next as an illustration.

*Example:* Consider a 3-channel pseudo-QMF design. The prototype filter, which is a linear-phase filter of length  $N = 36$ , is obtained by optimization. It has stopband attenuation  $A_s = 51.1$  dB and its stopband edge  $\omega_s = 0.296\pi$  rad. Each entry of the  $3 \times 3$  matrix  $\mathbf{E}(z)$  is a polynomial of length 12. The product  $\tilde{\mathbf{E}}(z)\mathbf{E}(z)$  was computed, which is also a  $3 \times 3$  matrix whose elements are polynomials of length 23. The entries of the first row of this product are shown in Table I. (In order to save space, the second and third rows are not shown.) Hence, it can be verified that the matrix  $\mathbf{E}(z)$  of this pseudo-QMF design is approximately lossless.

Three other related results are:

1) If the matrix  $\mathbf{E}(z)$  of a pseudo-QMF bank is lossless, then the pseudo-QMF bank (whose filters satisfy (4)) will necessarily satisfy the PR property (by using Lemma 3.1 [4]).

2) In [10]–[12], it is shown that  $M$ -channel pseudo-QMF banks can satisfy the PR property, when the length of the linear-phase prototype filter  $N$  is constrained to be  $N = 2M$ .

3) In [13], it is shown that the above result can be extended for the case when the prototype filter has length  $N = 2mM$ , where  $m$  is arbitrary.

The first result gives the condition that must be satisfied for pseudo-QMF banks to satisfy PR and raises the question, ‘‘Is it possible for the matrix  $\mathbf{E}(z)$  of a pseudo-QMF bank to be lossless?’’ The latter two results show that this is indeed possible (when the length of the prototype is  $N = 2mM$ , where  $m \geq 1$ ). In this paper, we present an approach to design cosine-modulated filter banks satisfying the PR property. The design procedure is based on a necessary and sufficient condition for the losslessness of the polyphase component matrix of the modulated filter bank. This condition is equivalent to the necessary and sufficient condition presented in [13]. The first step is to obtain a polyphase component representation of the cosine-modulated filter banks.

### C. Polyphase Implementation of Modulated Filter Banks

If we denote

$$c_{k,l} \triangleq 2 \cos \left( (2k+1) \frac{\pi}{2M} \left( l - \frac{N-1}{2} \right) + (-1)^k \frac{\pi}{4} \right)$$

then, using the periodicity of the cosine modulation, we get the relation

$$c_{k,(l+2pM)} = (-1)^p c_{k,l} \quad (7)$$

From this point onwards, the length of the prototype filter ( $N$ ) will be assumed an even multiple of  $M$ , i.e.,  $N =$

TABLE I  
EXAMPLE IN SECTION II-B. THE FIRST ROW OF  $\hat{E}(z)E(z)$  OF A 3-CHANNEL PSEUDO-QMF DESIGN

$[\hat{E}(z)E(z)]_{0,0}$	$[\hat{E}(z)E(z)]_{0,1}$	$[\hat{E}(z)E(z)]_{0,2}$
0.93968 D - 21	-0.13235 D - 22	-0.89997 D - 21
-0.69590 D - 05	-0.23399 D - 19	0.59822 D - 20
0.15585 D - 18	-0.63951 D - 19	0.81315 D - 19
0.21736 D - 03	0.69457 D - 18	-0.77927 D - 19
0.82339 D - 17	-0.27376 D - 17	-0.67593 D - 18
0.60492 D - 04	0.56921 D - 17	-0.35779 D - 17
0.85869 D - 16	-0.35020 D - 16	0.19516 D - 17
-0.70648 D - 04	0.43368 D - 16	-0.36429 D - 16
0.11102 D - 15	-0.29490 D - 16	-0.46838 D - 16
0.68909 D - 04	0.20470 D - 15	-0.79797 D - 16
-0.27756 D - 16	0.31225 D - 16	0.11796 D - 15
0.99994 D + 00	0.19429 D - 15	0.00000 D + 00
-0.27756 D - 16	-0.83267 D - 16	0.22204 D - 15
0.68909 D - 04	0.18041 D - 15	-0.15266 D - 15
0.11102 D - 15	-0.48572 D - 16	-0.58981 D - 16
-0.70648 D - 04	0.58981 D - 16	-0.69389 D - 16
0.85869 D - 16	-0.79797 D - 16	0.14745 D - 16
0.60492 D - 04	0.47705 D - 17	-0.32960 D - 16
0.82339 D - 17	-0.10734 D - 16	0.43368 D - 17
0.21736 D - 03	-0.40658 D - 19	-0.20600 D - 17
0.15585 D - 18	-0.76572 D - 18	0.93512 D - 18
-0.69590 D - 05	-0.34305 D - 19	0.10164 D - 18
0.93968 D - 21	0.38066 D - 21	-0.13764 D - 20

$2mM$ , where  $m$  is any positive integer (because in this paper, we will be dealing solely with prototypes satisfying this length constraint).

We shall now obtain a polyphase structure for the analysis filter bank. For this, first we express the prototype  $H(z)$  as

$$H(z) = \sum_{q=0}^{2M-1} \sum_{p=0}^{m-1} h(q + 2pM) z^{-(q+2pM)}$$

$$= \sum_{q=0}^{2M-1} z^{-q} G_q(z^{2M}) \quad (8)$$

where  $G_q(z)$  are the type 1 polyphase components [2] of  $H(z)$ . Using (1), the analysis filters can then be expressed as

$$H_k(z) = \sum_{n=0}^{N-1} h_k(n) z^{-n} = \sum_{n=0}^{2mM-1} h(n) c_{k,n} z^{-n}$$

$$= \sum_{q=0}^{2M-1} \sum_{p=0}^{m-1} h(q + 2pM) c_{k,(q+2pM)} z^{-(q+2pM)}. \quad (9)$$

Using (7), we can simplify (9) as

$$H_k(z) = \sum_{q=0}^{2M-1} z^{-q} c_{k,q} \sum_{p=0}^{m-1} (-1)^p h(q + 2pM) z^{-2pM}$$

$$= \sum_{q=0}^{2M-1} c_{k,q} z^{-q} G_q(-z^{2M}). \quad (10)$$

The analysis filter bank can be expressed in matrix form as

TABLE II  
COMPARISON OF MODULATED PR DESIGNS WITH PSEUDO-QMF AND LOSSLESS LATTICE BASED PR-QMF DESIGNS

Comparison Feature	Modulated PR Bank	Pseudo-QMF	Lattice PR-QMF
Distortions eliminated			
a) Magnitude	yes	no	yes
b) Phase	yes	yes	yes
c) Aliasing	yes	no	yes
Design			
a) Low complexity	yes	yes	no
b) Few parameters	yes	yes	no
Efficient Implementation	yes	yes	yes

$$\mathbf{h}(z) = \begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = \hat{\mathbf{C}} \begin{bmatrix} G_0(-z^{2M}) \\ z^{-1} G_1(-z^{2M}) \\ \vdots \\ z^{-(2M-1)} G_{2M-1}(-z^{2M}) \end{bmatrix} \quad (11)$$

where  $\hat{\mathbf{C}}$  is a  $M \times 2M$  cosine-modulation matrix and  $[\hat{\mathbf{C}}]_{k,l} = c_{k,l}$ ,  $0 \leq k \leq M-1$ ,  $0 \leq l \leq 2M-1$ . This implementation of the analysis filter bank (11) is shown in Fig. 1(c). An efficient implementation of  $\hat{\mathbf{C}}$  will be derived in Section IV-F. Using the relationship in (4), a similar implementation can be obtained for the synthesis filter bank. Equation (11) can be compactly expressed as

$$\mathbf{h}(z) = \hat{\mathbf{C}} \begin{bmatrix} \mathbf{g}_0(z^{2M}) & \mathbf{0} \\ \mathbf{0} & \mathbf{g}_1(z^{2M}) \end{bmatrix} \begin{bmatrix} \mathbf{e}_M(z) \\ z^{-M} \mathbf{e}_M(z) \end{bmatrix} \quad (12)$$

where  $\mathbf{e}_M^T(z) = [1 \ z^{-1} \ \cdots \ z^{-(M-1)}]$ , and  $\mathbf{g}_0(z)$ ,  $\mathbf{g}_1(z)$  are  $M \times M$  matrices defined as

$$\mathbf{g}_0(z) \triangleq \text{diag} [G_0(-z), G_1(-z), \dots, G_{M-1}(-z)]$$

$$\mathbf{g}_1(z) \triangleq \text{diag} [G_M(-z), G_{M+1}(-z), \dots, G_{2M-1}(-z)]. \quad (13)$$

The notation introduced in this section will be used in deriving PR filter banks (which use the same modulation as in pseudo-QMF banks). It must be mentioned that, in pseudo-QMF banks, the AC constraint ensures the cancellation of the significant aliasing terms, but there will always be a residual aliasing error due to the uncanceled aliasing terms. In the same manner, the flatness constraint minimizes the overall distortion (between the output and the input), but does not eliminate it. On the other hand, in PR filter banks, both these errors are completely eliminated. These facts are indicated in Table II, which highlights the differences between pseudo-QMF banks and PR filter banks.

### III. MODULATED PR FILTER BANKS WITH $N = 2mM$ , $m \geq 1$

In this section, we derive the conditions on a linear-phase prototype  $H(z)$  (length  $N = 2mM$ , where  $m$  is an arbitrary positive integer) such that the  $M$ -channel mod-

ulated filter bank satisfies the PR property. It must be noted that though  $H(z)$  has linear phase, the modulated filters do not have linear phase. Substituting  $N = 2mM$  in the expression for  $c_{k,l}$  (defined in Section I), we get

$$c_{k,l} = 2 \cos \left( (2k+1) \frac{\pi}{2M} \left( l - mM + \frac{1}{2} \right) + (-1)^k \frac{\pi}{4} \right). \quad (14)$$

The matrix  $\hat{C}$  (in (11)) can be expressed as  $\hat{C} = [A'_0 \ A'_1]$ , where  $A'_0, A'_1$  are given by

$$[A'_0]_{k,l} = c_{k,l} \quad \text{and} \quad [A'_1]_{k,l} = c_{k(l+M)}, \quad 0 \leq k, l \leq M-1. \quad (15)$$

The analysis filter bank (12) can then be written in terms of  $A'_0$  and  $A'_1$  as

$$\mathbf{h}(z) = \underbrace{[A'_0 \mathbf{g}_0(z^{2M}) + z^{-M} A'_1 \mathbf{g}_1(z^{2M})]}_{E(z^M)} \mathbf{e}_M(z) \quad (16)$$

where  $E(z)$  is the polyphase component matrix of the analysis filter bank.

Note that  $A'_0$  and  $A'_1$  depend on the value of  $m$ . In Appendix A it has been proved that these matrices satisfy the following properties (for all values of  $m$ ):

$$A_0'^{\dagger} A'_0 = 2M[\mathbf{I}_M + (-1)^{(m-1)} \mathbf{J}_M] \quad (17)$$

$$A_1'^{\dagger} A'_1 = 2M[\mathbf{I}_M - (-1)^{(m-1)} \mathbf{J}_M] \quad (18)$$

$$A_1'^{\dagger} A'_0 = A_0'^{\dagger} A'_1 = \mathbf{0} \quad (19)$$

where  $\mathbf{I}_M$  is the identity matrix and  $\mathbf{J}_M$  is the "reverse operator," defined in the introduction.

From [4, lemma 3.1], we know that if  $E(z)$ , the polyphase component matrix of the analysis bank is lossless, i.e.,  $\tilde{E}(z)E(z) = \mathbf{I}_M$ , then we can always find a synthesis bank such that the overall analysis/synthesis system satisfies perfect reconstruction (PR). So our aim is to obtain the conditions under which  $E(z)$ , the polyphase component matrix of the modulated filter bank, is lossless. We will now prove the following lemma.

*Lemma 1:* Let  $\mathbf{h}(z)$  be the analysis filter bank (11) obtained from  $H(z)$ , a real coefficient, linear-phase prototype filter of length  $N = 2mM$  ( $m \geq 1$ ). Then,  $E(z)$ , the polyphase component matrix of  $\mathbf{h}(z)$  is lossless if and only if

$$\tilde{G}_k(z)G_k(z) + \tilde{G}_{M+k}(z)G_{M+k}(z) = \frac{1}{2M}, \quad 0 \leq k \leq M-1 \quad (20)$$

where  $G_k(z)$  are the type 1 polyphase components [2] of  $H(z)$ .  $\diamond$

*Proof:* From (16) and (19), we can write

$$\tilde{E}(z)E(z) = \tilde{\mathbf{g}}_0(z^2)A_0'^{\dagger}A'_0\mathbf{g}_0(z^2) + \tilde{\mathbf{g}}_1(z^2)A_1'^{\dagger}A'_1\mathbf{g}_1(z^2). \quad (21)$$

Since the prototype  $H(z)$  is linear phase and its length is  $N = 2mM$ , we have the following relation [14] between the polyphase components of  $H(z)$ :

$$G_k(z) = z^{-(m-1)} \tilde{G}_{2M-1-k}(z), \quad 0 \leq k \leq M-1. \quad (22)$$

Using (22) we get,  $\mathbf{J}_M \mathbf{g}_i(z^2) = (-1)^{(m-1)} z^{-2(m-1)} \tilde{\mathbf{g}}_{(1-i)}(z^2) \mathbf{J}_M$ ,  $i = 0, 1$ , which in turn yields the result

$$\begin{aligned} \tilde{\mathbf{g}}_0(z^2) \mathbf{J}_M \mathbf{g}_0(z^2) &= (-1)^{(m-1)} z^{-2(m-1)} \tilde{\mathbf{g}}_0(z^2) \tilde{\mathbf{g}}_1(z^2) \mathbf{J}_M \\ &= \tilde{\mathbf{g}}_1(z^2) \mathbf{J}_M \mathbf{g}_1(z^2). \end{aligned} \quad (23)$$

Substituting (17), (18) in (21) and using (23), we get

$$\begin{aligned} \tilde{E}(z)E(z) &= 2M[\tilde{\mathbf{g}}_0(z^2)\mathbf{g}_0(z^2) + \tilde{\mathbf{g}}_1(z^2)\mathbf{g}_1(z^2)] \\ &\quad + (-1)^{(m-1)} 2M \underbrace{[\tilde{\mathbf{g}}_0(z^2)\mathbf{J}_M\mathbf{g}_0(z^2) - \tilde{\mathbf{g}}_1(z^2)\mathbf{J}_M\mathbf{g}_1(z^2)]}_{=0} \\ &= 2M[\tilde{\mathbf{g}}_0(z^2)\mathbf{g}_0(z^2) + \tilde{\mathbf{g}}_1(z^2)\mathbf{g}_1(z^2)]. \end{aligned} \quad (24)$$

From (24) we get

$$\begin{aligned} E(z) \text{ is lossless} \\ \Leftrightarrow 2M[\tilde{\mathbf{g}}_0(z^2)\mathbf{g}_0(z^2) + \tilde{\mathbf{g}}_1(z^2)\mathbf{g}_1(z^2)] = \mathbf{I}_M. \end{aligned} \quad (25)$$

The matrix equation (25) can be rewritten as  $M$  scalar equations (and since (25) holds for all values of  $z$ , we can replace  $-z^2$  by  $z$ ), which are precisely the conditions in (20). Thus, the lossless property of  $E(z)$  has been shown to be equivalent to the much simpler power-complementary condition in (20). This result can be summarized as "E(z) is lossless if and only if appropriate pairs of polyphase components of  $H(z)$  are power complementary."  $\nabla \nabla \nabla$

Note that it can be verified that the condition of the above Lemma as expressed in (20) is the Z-domain equivalent of the necessary and sufficient condition given by [13, eq. (11)].

The above Lemma covers all cosine-modulated filter banks which are derived from a linear-phase prototype, whose length  $N = 2mM$ . Owing to the linear-phase symmetry of  $H(z)$ , approximately half of the  $M$  constraints given in (20) are redundant. For example, using (22) it can be verified that the condition in (20) for  $k = k_1$ , where  $0 \leq k_1 \leq (M-1)$ , is the same as the condition for  $k = M-1-k_1$ . Removing the redundant constraints, (20) can be expressed as

1) For  $M$  even

$$\begin{aligned} \tilde{G}_k(z)G_k(z) + \tilde{G}_{M+k}(z)G_{M+k}(z) &= \frac{1}{2M}, \\ 0 \leq k \leq \frac{M}{2} - 1. \end{aligned} \quad (26)$$

2) For  $M$  odd

$$\begin{aligned} \tilde{G}_k(z) G_k(z) + \tilde{G}_{M+k}(z) G_{M+k}(z) &= \frac{1}{2M}, \\ 0 \leq k &\leq \left\lfloor \frac{M}{2} \right\rfloor - 1 \\ 2\tilde{G}_{(M-1)/2}(z) G_{(M-1)/2}(z) &= \frac{1}{2M}. \end{aligned} \quad (27)$$

The total number of independent constraints is  $\lfloor M/2 \rfloor$ , for  $M$  even/odd. From (27), we see that for  $M$  odd, the polyphase component  $G_{(M-1)/2}(z)$  is forced to be a pure delay. However, the value of the delay is not arbitrary. Since  $G_{(M-1)/2}(z)$  is a polyphase component of  $H(z)$ , which is a low-pass filter, the value of the delay is uniquely determined by the length of  $H(z)$ . By symmetry, the polyphase component  $G_{M+(M-1)/2}(z)$  is also a pure delay. A simple procedure to obtain  $G_{(M-1)/2}(z)$  and  $G_{M+(M-1)/2}(z)$  is mentioned in design example 1.

Further, for the special case with  $N = 2M$ , all the  $2M$  polyphase components are constants, i.e.,  $G_k(z) = h(k)$ ,  $0 \leq k \leq 2M - 1$ . If  $H(z)$  has real coefficients, the linear-phase symmetry yields  $h(k) = h(2M - 1 - k)$ . So we can express (20) as

$$\begin{aligned} h^2(k) + h^2(M+k) &= h^2(k) + h^2(M-1-k) = \frac{1}{2M}, \\ 0 \leq k &\leq M-1 \end{aligned} \quad (28)$$

which is exactly the condition obtained in [10]–[12]. If  $H(z)$  has complex coefficients satisfying  $h(n) = h^*(N - 1 - n)$ ,  $\forall n$ , then (28) becomes

$$|h(k)|^2 + |h(M-1-k)|^2 = \frac{1}{2M}, \quad 0 \leq k \leq M-1. \quad (29)$$

For the rest of this paper, we will consider prototypes with real coefficients only. The following result, which can be readily verified, will be used later.

*Fact 1:* Consider a linear-phase prototype of length  $N = 2mM$  whose impulse response coefficients are

$$h(k) = \begin{cases} \frac{1}{\sqrt{4M}}, & (mM - M) \leq k \leq (mM + M - 1) \\ 0, & \text{otherwise.} \end{cases} \quad (30)$$

This prototype satisfies the conditions given in (20).  $\diamond$

#### IV. DESIGN OF PROTOTYPE ( $N = 2mM$ )

In this section we focus on the design of the prototype (length  $N = 2mM$ ) for modulated filter banks satisfying the PR property. The approach is to obtain a prototype filter  $H(z)$  satisfying the conditions in (26), (27) which, by Lemma 1, are sufficient to ensure PR for the overall system. Further,  $H(z)$  should have high stopband atten-

uation and a narrow transition bandwidth. So,  $H(z)$  must be obtained by optimization.

One way of satisfying the conditions in (26), (27) during optimization is via spectral factorization. Suppose  $G_k(z)$ , one of the polyphase components of the power complementary pair  $\{G_k(z), G_{M+k}(z)\}$  is optimized, then  $G_{M+k}(z)$  can be computed by spectral factorization. This must be done for each of the power complementary pairs, and the same process must be repeated in every iteration (during the optimization). This would then amount to  $\lfloor M/2 \rfloor$  spectral factor computations per iteration. However, there is a way to completely avoid this extensive amount of computation. This is achieved by using the two-channel lossless lattice which is discussed next. We will fully exploit the advantage of these lattices in our design approach.

##### A. Two-Channel Lossless Lattice

We have the following result from [22], which introduces the two-channel lossless lattice.

*Fact 2:* A stable digital filter transfer function  $P(z)$  with real coefficients is said to be bounded real (BR) if  $|P(e^{j\omega})| \leq 1$ ,  $\forall \omega$ . And any FIR BR pair  $\{P(z), Q(z)\}$  satisfying

$$\tilde{P}(z)P(z) + \tilde{Q}(z)Q(z) = 1, \quad \forall z \quad (31)$$

can always be realized as a nonrecursive, cascaded, two-channel lossless lattice structure shown in Figs. 2(a), (b).  $\diamond$

In Fig. 2(a), the two-channel lossless lattice is made up of a cascade of normalized, four-multiplier lattice sections. Each lattice section is characterized by one parameter  $\theta_j$ , where the index  $j$  refers to the particular lattice section. In Fig. 2(b), the lattice is made up of a cascade of the denormalized, two-multiplier lattice sections characterized by the parameter  $\beta_j$  and the overall scaling multiplier  $\alpha$  which is defined as  $\alpha \triangleq \prod_{j=0}^q 1/\sqrt{1 + \beta_j^2}$ .

In this paper, we use the structure of Fig. 2(a) in obtaining the design procedure. However, since the two structures are equivalent [22], the design procedure can be readily translated to the structure of Fig. 2(b). Both lattices have the same number of parameters but the latter is used in the implementation of the filter bank since it requires approximately half the number of multipliers as the former.

##### B. Satisfying the Pairwise Power Complementary Property

From Lemma 1, we have the condition that requires pairs of polyphase components to satisfy the power complementary (PC) property. From Fact 2, we see that each of these pairs  $\{G_k(z), G_{M+k}(z)\}$ , can be designed (to within a scale factor) by using a separate two-channel lossless lattice. During the optimization, we work directly with the lattice parameters. Hence the polyphase component pairs are guaranteed to satisfy the pairwise PC property which, in turn, ensures PR. The same is true even in the presence of coefficient quantization. So, in other

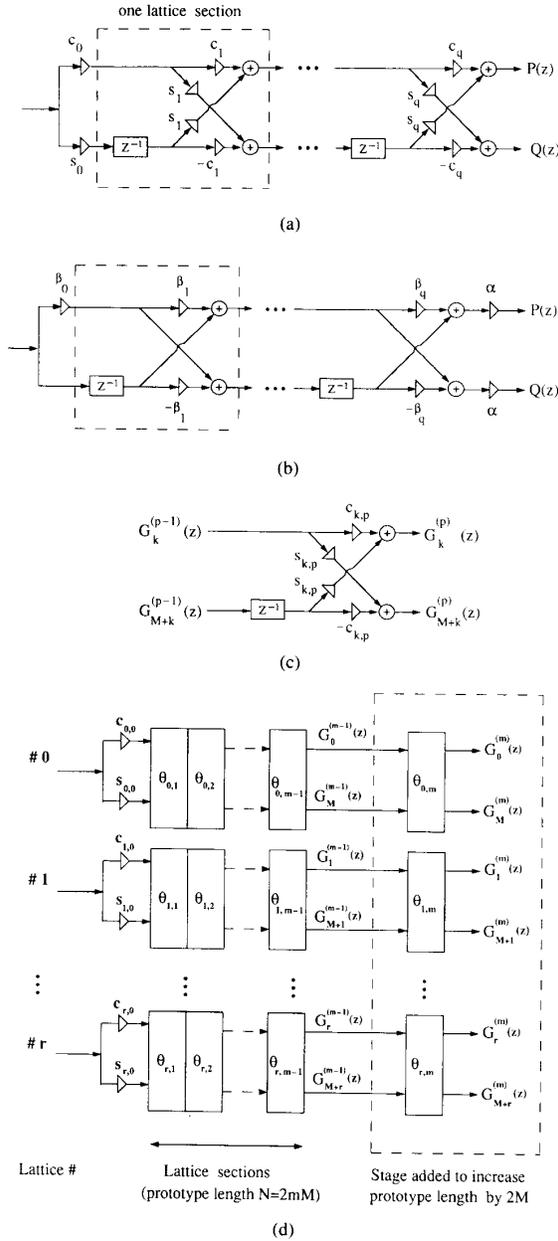


Fig. 2. (a) The two-channel lossless lattice with four-multiplier lattice sections ( $c_k = \cos \theta_k$  and  $s_k = \sin \theta_k$ ). (b) The two-channel lossless lattice with two-multiplier lattice sections.  $\alpha$  is the scaling multiplier. (c) Typical four-multiplier implementation of the  $p$ th lattice section of the  $k$ th lattice. (d) A block diagram of the lattices used in the design of a  $M$ -channel prototype  $H(z)$ . The total number of lattices =  $r + 1 = \lfloor M/2 \rfloor$ .

words, the advantage of using the two-channel lossless lattices is that the PR condition is inherently satisfied and hence, it need not be included as one of the constraints in the design of  $H(z)$ .

For  $M$  even (refer to (26)), we design  $M/2$  pairs of polyphase components, with the rest being determined by symmetry. For  $M$  odd (refer to (27)), we design  $\lfloor M/2 \rfloor$

pairs of polyphase components, with  $G_{(M-1)/2}(z)$ ,  $G_{M+(M-1)/2}(z)$  being forced to be pure delays and the remaining  $\lfloor M/2 \rfloor$  pairs of polyphase components being determined by symmetry. So, for any  $M$ , the number of two-channel lossless lattices needed for the design of the prototype is  $\lfloor M/2 \rfloor$ .

In Fig. 2(d), we have a block diagram representation of the  $\lfloor M/2 \rfloor$  two-channel lattices. The  $k$ th lattice yields the PC pair  $\{G_k(z), G_{M+k}(z)\}$ . Its parameters are denoted as  $\theta_{k,j}$ , where the index  $j$  refers to the particular lattice section. The transfer function between the input of the lattice and the output of the  $p$ th lattice section is denoted by a superscript  $p$ . The lattice transfer functions are initialized as

$$G_k^{(0)}(z) = \cos \theta_{k,0} \quad \text{and} \quad G_{M+k}^{(0)}(z) = \sin \theta_{k,0},$$

$$0 \leq k \leq \left\lfloor \frac{M}{2} \right\rfloor - 1. \quad (32)$$

In Fig. 2(c), we have a typical four-multiplier implementation of the  $p$ th lattice section of the  $k$ th lattice. Let  $\{G_k^{(p-1)}(z), G_{M+k}^{(p-1)}(z)\}$  be the transfer functions from the input to the output of the  $(p-1)$ th section of the  $k$ th lattice. We can then write  $\{G_k^{(p)}(z), G_{M+k}^{(p)}(z)\}$  as

$$\begin{bmatrix} G_k^{(p)}(z) \\ G_{M+k}^{(p)}(z) \end{bmatrix} = \begin{bmatrix} \cos \theta_{k,p} & \sin \theta_{k,p} \\ \sin \theta_{k,p} & -\cos \theta_{k,p} \end{bmatrix} \begin{bmatrix} G_k^{(p-1)}(z) \\ z^{-1} G_{M+k}^{(p-1)}(z) \end{bmatrix},$$

$$p \geq 1, 0 \leq k \leq \left\lfloor \frac{M}{2} \right\rfloor - 1. \quad (33)$$

Hence we get a recursive relation for the transfer function when a new lattice section is added. From (33), it can be seen that the addition of each lattice section increases the order of the transfer functions by one.

1) *Number of Parameters to be Optimized:* From the preceding discussion, we know that  $\lfloor M/2 \rfloor$  two-channel lattices are used in the design of the prototype  $H(z)$  for a  $M$ -channel modulated PR filter bank. If the length of the prototype is  $N = 2mM$ , then each of  $2M$  polyphase components has length  $m$  (and hence, order  $(m-1)$ ). This implies that each lattice has  $(m-1)$  sections, involving a total of  $m$  unknown parameters  $[\theta_{k,0}, \theta_{k,1}, \dots, \theta_{k,(m-1)}]$ . Hence the total number of parameters to be optimized is  $m \lfloor M/2 \rfloor$ .

On the other hand, a pseudo-QMF design [16]–[21] (same length prototype) requires  $mM$  parameters while the lossless lattice approach to PR-QMF design [4], [8], [15] requires many more, viz.,  $[(2m-1)(M-1) + M(M-1)/2]$ . A comparison of the number of parameters to be optimized in each of the three design approaches—the modulated PR bank, pseudo-QMF, and lossless lattice based PR-QMF banks, is shown in Table III.

From Table III, we see that modulated PR filter bank approach requires approximately half the number of parameters as the pseudo-QMF design and much fewer parameters when compared with the traditional PR-QMF approach. This advantage becomes significant, particu-

TABLE III  
COMPARISON OF THE NUMBER OF PARAMETERS OPTIMIZED IN THE DESIGN OF MODULATED PR BANKS, PSEUDO-QMF BANKS, AND LOSSLESS LATTICE BASED PR-QMF BANKS

# channels $M$	Length $N$	# of Parameters Optimized		
		Modulated PR Banks	Pseudo-QMF	Lattice PR-QMF
3-channels	48	8	24	33
	60	10	30	41
5-channels	40	8	20	38
	60	12	30	54
7-channels	42	9	21	51
	84	18	42	87
16-channels	64	16	32	165
	96	24	48	195
17-channels	68	16	34	184
	102	24	51	216

larly for large  $M$ . In the next section (in design example 2), we will present design comparisons between the prototypes of modulated PR banks and pseudo-QMF banks.

### C. Design Steps

The procedure for designing the  $M$ -channel prototype filter (length  $N = 2mM$ ) involves the initialization of the parameters of the  $\lfloor M/2 \rfloor$  lattices and the optimization of these parameters. We discuss each aspect separately.

1) *Initialization*: The following is a simple initialization scheme for all the parameters of the  $\lfloor M/2 \rfloor$  two-channel lattices (each two-channel lattice has  $(m - 1)$  sections)

$$\theta_{k,p} = \begin{cases} \frac{\pi}{4}, & p = 0, & 0 \leq k \leq \lfloor \frac{M}{2} \rfloor - 1 \\ \frac{\pi}{2}, & 1 \leq p \leq (m - 1), & 0 \leq k \leq \lfloor \frac{M}{2} \rfloor - 1. \end{cases} \quad (34)$$

It can be verified that this initialization corresponds to a prototype  $H(z)$  with

$$h(k) = \begin{cases} 1, & (mM - M) \leq k \leq (mM + M - 1) \\ 0, & \text{otherwise.} \end{cases} \quad (35)$$

With appropriate scaling, this prototype satisfies (26), (27) (as mentioned earlier in Fact 1). This prototype  $H(z)$  has stopband attenuation  $A_s \approx 13$  dB and stopband edge  $\omega_s < \pi/M$  rad. This approach is independent of the value of  $m$  (i.e., independent of  $N$ ) and from (35) we see that exactly  $2M$  coefficients of  $H(z)$  are initialized to be non-zero while the remaining  $(2mM - 2M)$  coefficients are set to zero. As a result, this scheme works well for smaller values of  $m$  whereas for larger values of  $m$ , a different approach (which is described in Section IV-E) works better.

2) *Optimization*: Having initialized all  $m \lfloor M/2 \rfloor$  parameters, they are then optimized using standard optimization routines (e.g., E04JAF [27], based on the quasi-Newton algorithm) to minimize the objective function, either  $\Phi_1$  or  $\Phi_2$  given below:

$$\Phi_1 = \int_{(\pi/2M) + \delta}^{\pi} |H(e^{j\omega})|^2 d\omega, \quad \Phi_2 = \max_{\omega \in [(\pi/2M) + \delta, \pi]} |H(e^{j\omega})| \quad (36)$$

where  $\delta < \pi/2M$ . Using  $\Phi_1$ , the problem involves the minimization of the stopband energy (yielding a minimum energy solution) while with  $\Phi_2$ , it involves the minimization of the maximum of the filter response in the stopband (yielding a minimax solution). In all the examples, it was observed that using  $\Phi_1$  produces quicker convergence whereas using  $\Phi_2$  gives a prototype with lower stopband attenuation  $A_s$  (nearly equiripple solution). In order to combine the advantages of both objective functions  $\Phi_1$ ,  $\Phi_2$ , the following heuristic scheme works well.

*Step 1*: After initialization, optimize using  $\Phi_1$  and obtain the minimum energy solution.

*Step 2*: Using the prototype obtained in step 1 as the starting point, run the optimization using  $\Phi_2$  and obtain the minimax solution.

For step 2, it was observed that the optimization can be terminated after approximately  $100 * m \lfloor M/2 \rfloor$  iterations. (Note that  $m \lfloor M/2 \rfloor =$  total number of parameters being optimized.) Doing step 2 is optional. However, in most cases, the prototype obtained by doing step 2 after step 1 had higher stopband attenuation than the prototype at the end of step 1. All the above-mentioned features are demonstrated in the design examples in Section V.

### D. Increasing the Length of the Prototype

Another attractive feature of this design approach is the ease with which a prototype  $H(z)$  of a particular length  $N = 2mM$  can be used to obtain a prototype  $H'(z)$  of longer length (increments in length are in multiples of  $2M$ ). Let the length of  $H'(z)$  be  $2mM + 2M$ . The increase in the length of the prototype by  $2M$  directly translates into a unit increase in the lengths of each of the  $2M$  polyphase components. This implies that precisely one lattice section must be added to each of the  $\lfloor M/2 \rfloor$  two-channel lattices (which are used in the design of the prototype). In this sense, the structure has a hierarchical property. This is shown schematically in Fig. 2(d). The procedure to obtain  $H'(z)$  is as follows. First, the values of all the  $m \lfloor M/2 \rfloor$  lattice parameters used to obtain  $H(z)$  are retained. Then, for the newly added lattice sections, we set  $\theta_{k,m} = \pi/2, \forall k$ . Finally, all the  $(m + 1) \lfloor M/2 \rfloor$  parameters are optimized to yields the new prototype  $H'(z)$ .

### E. Two Stage Design of Long Length Prototypes

The above feature, which allows the length of the prototype to be increased, also gives an approach to design prototypes of long length (i.e., large values of  $m$ ). Let the

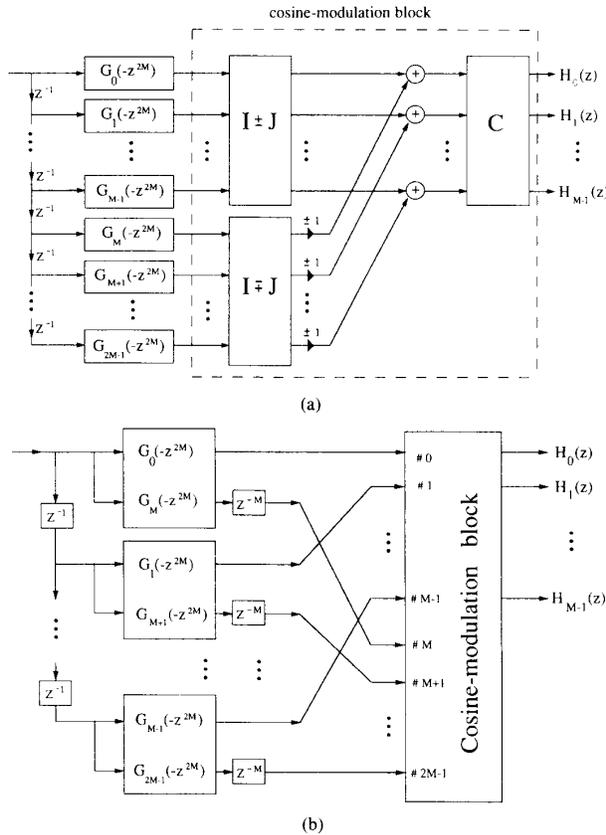


Fig. 3. (a) Implementation of a cosine-modulated filter bank derived from a linear-phase prototype filter of length  $N = 2mM$  (including pseudo-QMF banks).  $C$  is the type IV discrete cosine transform (DCT). (b) Implementation of cosine-modulated PR analysis filter bank. Each polyphase component pair  $\{G_k(-z^{2M}), G_{M+k}(-z^{2M})\}$  is implemented by a two-channel lossless lattice. (A total of  $M$  lattices are used.) The cosine-modulation block is given in Fig. 3(a).

desired length of the prototype be  $N_1 = 2m_1M$ , where  $m_1 > 3$ . The design steps are summarized below.

a) Design a prototype for  $m = 2$  or  $3$  by doing step 1 of the optimization. Then use this as an initialization for the next design.

b) Add  $(m_1 - m)$  additional sections to each of the  $\lfloor M/2 \rfloor$  lattices used in the design and initialize the parameters of these added sections by setting them to  $\pi/2$ .

c) Optimize (using step 1) to obtain the desired prototype filter.

d) Rerun the optimization (using step 2), if needed.

The above two stage approach has been extensively tested and has been found to work well.

#### F. Implementation and Complexity of Modulated PR Banks

We will now obtain an efficient implementation which is applicable to cosine-modulated filter banks satisfying (1) obtained from a linear-phase prototype of length  $N = 2mM$ . (There are no restrictions on  $M$  or  $m$ .)

TABLE IV  
COMPARISON OF IMPLEMENTATION COMPLEXITY OF MODULATED PR BANKS AND PSEUDO-QMF BANKS

		MPU	APU
Modulated PR Banks	$M$ even	$2(m+1)$	$2(m-1)$
	$M$ odd	$2\alpha(m+1)$	$2\alpha(m-1)$
Pseudo-QMF Banks		$2m$	$2(m-1)$

$$\text{Where } \alpha = \left(2 \frac{\lfloor M/2 \rfloor}{M}\right) < 1.$$

MPU (APU)—Multiplications (additions) per unit time.

The modulation matrix  $\hat{C}$  is expressed as  $\hat{C} = [A_0^T \ A_1^T]$  and using the relations in Table X along with the results in (A.10), (A.11), and (A.15), we have

$$\hat{C} = \begin{cases} \sqrt{M}(-1)^{m_1} C[(I - J) & -(I + J)] \\ \text{for } m \text{ even } (m = 2m_1) \\ \sqrt{M}(-1)^{m_1} C[(I + J) & (I - J)], \\ \text{for } m \text{ odd } (m = 2m_1 + 1), \end{cases} \quad (37)$$

where  $C$  is the type IV discrete cosine transform [25] whose definition is given in (A.6). Using (37) in (11), we get the implementation in Fig. 3(a) (where the constant scale factor  $\sqrt{M}(-1)^{m_1}$  has been omitted). For the special case when  $M$  is even and  $m$  is even, Fig. 3(a) can be further simplified to obtain the implementation in [17]. The complexity of the entire modulation section is  $3M$  adders along with the complexity of the DCT. Since the modulation part is identical for modulated PR banks and pseudo-QMF banks, in the following comparison, we will consider only the complexity of implementing the polyphase components  $[G_0(-z^2), G_1(-z^2) \cdots G_{2M-1}(-z^2)]$ .

In pseudo-QMF designs, the  $2M$  polyphase components (each of length  $m$ ) are implemented in direct form requiring  $2mM$  multipliers and  $2M(m-1)$  adders. For modulated PR banks, the  $2M$  polyphase components are implemented as  $2 \lfloor M/2 \rfloor$  two-channel lossless lattices (Fig. 3(b)). Using two-multiplier lattice sections, each lattice requires  $(2m+2)$  multipliers (including the two scaling multipliers) and  $2(m-1)$  adders. The multiplications per unit time (MPU) and additions per unit time (APU) are two measures of implementation complexity. The corresponding MPU and APU values for modulated PR banks and pseudo-QMF banks are given in Table IV.

#### V. EXAMPLES OF MODULATED PR FILTER BANKS

*Design Example 1:* This example demonstrates the different aspects of the new approach to design of the prototype filter  $H(z)$  of a modulated PR filter bank. We will consider the case  $M = 17$  channels. (As  $M$  is a prime number, the filter bank cannot be implemented as a tree structure.) Since  $\lfloor M/2 \rfloor = 8$ , we require 8 two-channel lossless lattices in order to design the eight pairs of power complementary (PC) polyphase components (which sat-

TABLE V  
 DESIGN EXAMPLE 1. DESIGN OF A 17-CHANNEL MODULATED PR BANK PROTOTYPE

Polyphase Components	Lattice Number							
	0	1	2	3	4	5	6	7
	$G_0(z), G_{17}(z)$	$G_1(z), G_{18}(z)$	$G_2(z), G_{19}(z)$	$G_3(z), G_{20}(z)$	$G_4(z), G_{21}(z)$	$G_5(z), G_{22}(z)$	$G_6(z), G_{23}(z)$	$G_7(z), G_{24}(z)$

isfy (27)). Table V shows the particular polyphase components designed by each of the eight lattices. Since  $M$  is odd, two of the polyphase components are forced to be pure delays (as mentioned in Section III). They are  $G_8(z)$  and  $G_{25}(z)$ , which are obtained by using a separate two-channel lattice with the parameters chosen as in (34). This ensures that these two polyphase components are pure delays (of appropriate values). The remaining eight pairs of polyphase components are obtained by symmetry relations (given in (22)), owing to the linear-phase property of the prototype.

The length of the prototype filter  $H(z)$  is  $N = 2mM$ . We now present three designs—for prototype of length  $N = 68$ ,  $N = 102$ , and  $N = 136$ . The corresponding values of  $m$  are 2, 3, and 4, respectively. The total number of parameters to be optimized in each design is  $8m(= m \lfloor M/2 \rfloor)$ . First we design the prototype with  $N = 68$ . In this design, the parameters are initialized as in (34). After optimization, we obtain the desired prototype filter. Then we increase the length (as explained in Section IV-D) to obtain prototypes of length  $N = 102$  and 136. In each case, after initialization of the lattice parameters, the design is completed by doing both Steps 1 and 2 of the optimization. For each of the three designs, the magnitude response of the prototype at the end of step 1 (minimum energy design) and at end of step 2 (Minimax design) are plotted in Figs. 4(a)–(c). To facilitate comparison, the minimum energy design is shown by a broken line while the minimax design is shown by a solid line. The corresponding values of stopband attenuation ( $A_s$ ) and the stopband edge ( $\omega_s$ ) for each design are presented in Table VI. Hence it can be readily seen that in each instance, doing step 2 of the optimization (i.e., rerunning the optimization with  $\Phi_2$  as the objective function) improved the  $A_s$  of the prototype. The impulse response coefficients of the prototype filter (with  $N = 102$ ) are given in Table VII. Its frequency response is shown in Fig. 5(a) and the responses of all the filters in the PR bank, obtained by the cosine-modulation of  $H(z)$  as in (1), are plotted in Fig. 5(b).

*Design Example 2:* In this example, we present a comparison between the modulated PR filter banks and pseudo-QMF banks. One of the differences between the two approaches must be mentioned at the outset. That is, in pseudo-QMF banks, an assumption is made that filters belonging to nonadjacent channels do not overlap (i.e., in a seven channel pseudo-QMF bank,  $H_3(e^{j\omega})$  has overlaps with  $H_2(e^{j\omega})$  and  $H_4(e^{j\omega})$ ). The passbands of all the other

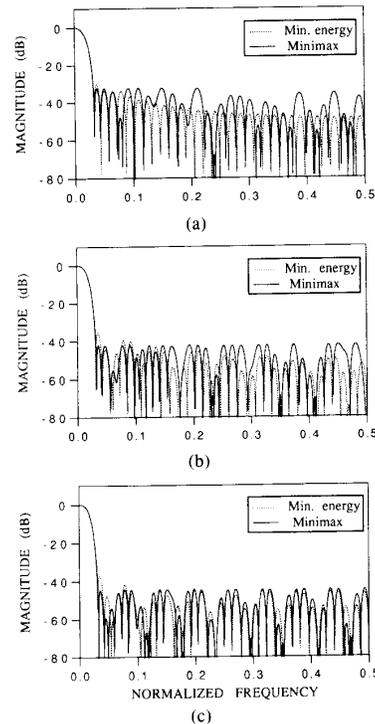

 Fig. 4. Design example 1. Magnitude responses of 17-channel prototype after step 1 (minimum energy) and step 2 (Minimax). (a)  $N = 68$ , (b)  $N = 102$ , and (c)  $N = 136$ .

 TABLE VI  
 DESIGN EXAMPLE 1. COMPARISON OF THE 17-CHANNEL MODULATED PR BANK PROTOTYPE AFTER STEPS 1 AND 2 OF THE OPTIMIZATION

Length $N$	Step 1 (Min. Energy)		Step 2 (Minimax)	
	$A_s$ (dB)	$\omega_s$ (Rad)	$A_s$ (dB)	$\omega_s$ (Rad)
68	30.51	$0.0644\pi$	32.45	$0.0644\pi$
102	35.72	$0.0620\pi$	42.16	$0.0644\pi$
136	37.22	$0.0614\pi$	44.51	$0.0644\pi$

filters lie in the stopband of  $H_3(e^{j\omega})$ . This places a constraint on the transition bandwidth of the prototype filter. Such a constraint is not necessary in the case of modulated PR banks. For purposes of comparison, we will look at prototypes (designed by both methods) with the same transition bandwidth.

The performance of a QMF bank is measured by following two quantitative criteria.

TABLE VII  
DESIGN EXAMPLE 1. IMPULSE RESPONSE COEFFICIENTS OF PROTOTYPE ( $N = 102$ ) OF A 17-CHANNEL MODULATED PR FILTER BANK

$n$	$h(n)$	$n$	$h(n)$	$n$	$h(n)$
0	-4.272049 E - 04	34	1.376675 E - 02	68	1.316380 E - 02
1	-4.853395 E - 04	35	1.633310 E - 02	69	1.082142 E - 02
2	-4.997748 E - 04	36	1.759948 E - 02	70	9.552895 E - 03
3	-5.129669 E - 04	37	2.034467 E - 02	71	7.256847 E - 03
4	-5.288108 E - 04	38	2.191992 E - 02	72	6.355320 E - 03
5	-3.490611 E - 04	39	2.431080 E - 02	73	3.784142 E - 03
6	-3.878599 E - 04	40	2.595075 E - 02	74	3.757966 E - 03
7	-2.579029 E - 04	41	2.742702 E - 02	75	3.160015 E - 03
8	1.105338 E - 34	42	2.948043 E - 02	76	-1.805155 E - 18
9	-2.180628 E - 03	43	3.116092 E - 02	77	-3.737350 E - 04
10	-1.960596 E - 03	44	3.234317 E - 02	78	-7.434289 E - 04
11	-1.714884 E - 03	45	3.360359 E - 02	79	-7.702541 E - 04
12	-1.928333 E - 03	46	3.478912 E - 02	80	-1.742832 E - 03
13	-1.959910 E - 03	47	3.555154 E - 02	81	-1.899333 E - 03
14	-2.102945 E - 03	48	3.643303 E - 02	82	-2.270290 E - 03
15	-2.075392 E - 03	49	3.665203 E - 02	83	-2.530636 E - 03
16	-2.265407 E - 03	50	3.693109 E - 02	84	-2.482397 E - 03
17	-2.482397 E - 03	51	3.693109 E - 02	85	-2.265407 E - 03
18	-2.530636 E - 03	52	3.665203 E - 02	86	-2.075392 E - 03
19	-2.270290 E - 03	53	3.643303 E - 02	87	-2.102945 E - 03
20	-1.899333 E - 03	54	3.555154 E - 02	88	-1.959910 E - 03
21	-1.742832 E - 03	55	3.478912 E - 02	89	-1.928333 E - 03
22	-7.702541 E - 04	56	3.360359 E - 02	90	-1.714884 E - 03
23	-7.434289 E - 04	57	3.234317 E - 02	91	-1.960596 E - 03
24	-3.737350 E - 04	58	3.116092 E - 02	92	-2.180628 E - 03
25	-1.805155 E - 18	59	2.948043 E - 02	93	1.105338 E - 34
26	3.160015 E - 03	60	2.742702 E - 02	94	-2.579029 E - 04
27	3.757966 E - 03	61	2.595075 E - 02	95	-3.878599 E - 04
28	3.784142 E - 03	62	2.431080 E - 02	96	-3.490611 E - 04
29	6.355320 E - 03	63	2.191992 E - 02	97	-5.288108 E - 04
30	7.256847 E - 03	64	2.034467 E - 02	98	-5.129669 E - 04
31	9.552895 E - 03	65	1.759948 E - 02	99	-4.997748 E - 04
32	1.082142 E - 02	66	1.633310 E - 02	100	-4.853395 E - 04
33	1.316380 E - 02	67	1.376675 E - 02	101	-4.272049 E - 04

TABLE VIII  
DESIGN EXAMPLE 2. COMPARISON BETWEEN MODULATED PR-BANKS AND PSEUDO-QMF BANKS—7 CHANNEL ( $N = 42$ )

	Prototype		Reconstruction Error ( $E_{p-p}$ )	Aliasing Error ( $E_a$ )
	$A_s$ (dB)	$\omega_s$ (Rad)		
Pseudo-QMF bank	39.03	0.1424 $\pi$	1.296 E - 03	8.848 E - 04
	38.55	0.1420 $\pi$	2.095 E - 04	9.198 E - 04
	38.14	0.1414 $\pi$	9.459 E - 05	1.022 E - 03
Modulated PR Bank	34.13	0.1426 $\pi$	1.998 E - 15	8.517 E - 16

2) *The Aliasing Error ( $E_a$ ):* The output of the analysis/synthesis system can be expressed in terms of the input as

$$\hat{X}(z) = \frac{1}{M} X(z) \sum_{k=0}^{M-1} H_k(z) F_k(z) + \frac{1}{M} \sum_{l=1}^{M-1} X(zW^l) \underbrace{\sum_{k=0}^{M-1} H_k(zW^l) F_k(z)}_{\text{alias terms}} \quad (39)$$

and the total aliasing error is given by

$$E(\omega) = \frac{1}{M} \left[ \sum_{l=1}^{M-1} |A_l(e^{j\omega})|^2 \right]^{1/2} \quad (40)$$

where  $A_l(z) = \sum_{k=0}^{M-1} H_k(zW^l) F_k(z)$  and  $E_a \triangleq \max_{\omega} E(\omega)$ . In the design comparisons, the stopband attenuation ( $A_s$ ) and stopband edge ( $\omega_s$ ) of each prototype (modulated PR and pseudo-QMF) are tabulated along with their respective  $E_{p-p}$  and  $E_a$  values, first for a 7-channel design and then for a 17-channel design.

In the 7-channel design, the length of the prototype is  $N = 42$  (the corresponding value of  $m$  is 3). For the modulated PR prototype, three lattices are used. The lattice parameters (total = 9) are initialized as in (34) and the prototype is obtained by doing both steps 1 and 2 of the optimization. The resulting prototype filter has stopband attenuation  $A_s = 34.13$  dB and stopband edge  $\omega_s = 0.1426\pi$  rad. The values of  $E_{p-p}$  and  $E_a$  are  $1.998 E - 15$  and  $8.517 E - 16$ , respectively. In pseudo-QMF designs, relative weighting is used in the objective function to trade off between the flatness constraint (affecting  $E_{p-p}$ ) and the prototype filter stopband energy (affecting  $E_a$ ). The three designs are obtained with different values of relative weights. The parameters of all the designs are shown in Table VIII.

In the 17-channel comparison, the modulated PR prototype has  $N = 102$  with  $A_s = 35.72$  dB and  $\omega_s = 0.0586\pi$  rad. Its values of  $E_{p-p}$  and  $E_a$  are  $8.216 E - 15$  and  $1.041 E - 15$ , respectively. As in the previous comparison, the modulated PR prototype is compared with three different pseudo-QMF prototypes and the results are given in Table IX. In both the above design examples, we see that approximately 5 dB (in stopband attenuation) is the price

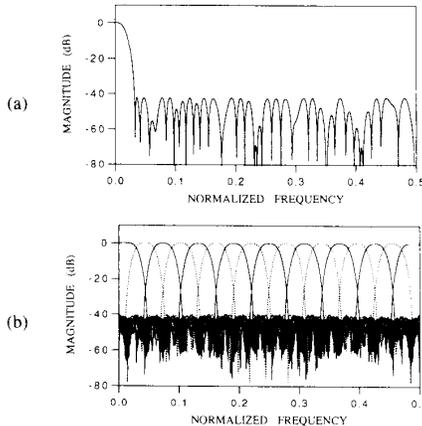


Fig. 5. Design example 1. (a) Response of the 17-channel prototype ( $N = 102$ ). (b) Seventeen-channel cosine-modulated PR filter bank.

1) *The Peak-to-Peak Reconstruction Error ( $E_{p-p}$ ):* The transfer function of the overall analysis/synthesis system is given by  $T(z) = (1/M) \sum_{k=0}^{M-1} H_k(z) F_k(z)$ . Using filters whose responses are normalized to unity, we get

$$(1 - \delta_1) \leq M|T(e^{j\omega})| \leq (1 + \delta_2) \quad (38)$$

and  $E_{p-p}$  is defined as  $E_{p-p} \triangleq \delta_1 + \delta_2$ .

TABLE IX  
DESIGN EXAMPLE 2. COMPARISON BETWEEN MODULATED PR-BANKS AND PSEUDO-QMF BANKS—17 CHANNEL ( $N = 102$ )

	Prototype		Reconstruction Error ( $E_{p-p}$ )	Aliasing Error ( $E_a$ )
	$A_s$ (dB)	$\omega_s$ (Rad)		
Pseudo-QMF bank	40.65	$0.0590\pi$	$6.790 \text{ E} - 03$	$3.794 \text{ E} - 04$
	38.68	$0.0585\pi$	$2.139 \text{ E} - 04$	$3.193 \text{ E} - 04$
	38.42	$0.0581\pi$	$8.749 \text{ E} - 05$	$8.113 \text{ E} - 04$
Modulated PR bank	35.72	$0.0586\pi$	$8.216 \text{ E} - 15$	$1.041 \text{ E} - 15$

paid to obtain PR. It must be mentioned that it is not possible to choose the relative weights (for the objective function) in pseudo-QMF designs such that either  $E_{p-p}$  or  $E_a$  can be made arbitrarily small or comparable to the corresponding values in the modulated PR designs.

## VI. CONCLUSION

In this paper, we have presented a derivation of the necessary and sufficient condition on the polyphase components of a linear-phase prototype (length  $N = 2mM$ , where  $m \geq 1$ ) such that  $\mathbf{E}(z)$ , the polyphase component matrix of the cosine-modulated analysis filter bank, is lossless. The losslessness of  $\mathbf{E}(z)$ , in turn, ensures that the analysis/synthesis system (using modulated filter banks for analysis and synthesis) satisfies the perfect reconstruction (PR) property. An efficient procedure to design the prototype (satisfying the above necessary and sufficient condition) is presented. This design procedure, based on the two-channel lossless lattice, involves fewer parameters to be optimized than pseudo-QMF designs and much fewer than lossless lattice based PR-QMF designs. This advantage becomes significant for large  $M$  (number of channels) and for long length prototypes. Further, the hierarchical structure of the lattice, which makes it flexible to increase the length of the prototype filter, is exploited in the design procedure. Using this approach, PR filter banks (FIR) can be designed for an arbitrary number of channels. Since both the analysis and synthesis filter banks are obtained by cosine-modulation, an efficient implementation is derived using the  $2M$  polyphase components of the prototype filter and the discrete cosine transform (DCT) matrix. The details of design procedure and complexity of implementation are discussed and all the above-mentioned aspects are demonstrated by the examples and detailed comparisons.

## APPENDIX A PROPERTIES OF MODULATED PR FILTER BANKS

In this Appendix, we present a proof for the results in (17)–(19). The proof is given in two parts.

*Part I:* Let  $\mathbf{A}_0$  and  $\mathbf{A}_1$  be equal to  $\mathbf{A}'_0$  and  $\mathbf{A}'_1$ , respectively, for the particular case when  $m = 1$ . Hence, using (14) and (15) we get

$$[\mathbf{A}_0]_{k,l} = 2 \cos \left( (2k+1) \frac{\pi}{2M} \left( l - M + \frac{1}{2} \right) + (-1)^k \frac{\pi}{4} \right), \quad 0 \leq k, l \leq M-1 \quad (\text{A.1})$$

$$[\mathbf{A}_1]_{k,l} = 2 \cos \left( (2k+1) \frac{\pi}{2M} \left( l + \frac{1}{2} \right) + (-1)^k \frac{\pi}{4} \right), \quad 0 \leq k, l \leq M-1. \quad (\text{A.2})$$

Now, we prove the following results:

$$\mathbf{A}_0^\dagger \mathbf{A}_0 = 2M[\mathbf{I}_M + \mathbf{J}_M] \quad (\text{A.3})$$

$$\mathbf{A}_1^\dagger \mathbf{A}_1 = 2M[\mathbf{I}_M - \mathbf{J}_M] \quad (\text{A.4})$$

$$\mathbf{A}_1^\dagger \mathbf{A}_0 = \mathbf{A}_0^\dagger \mathbf{A}_1 = \mathbf{0}. \quad (\text{A.5})$$

*Proof of (A.3)–(A.5):* Let  $\mathbf{C}$  and  $\mathbf{S}$  be the type IV discrete cosine transform (DCT) and type IV discrete sine transform (DST) [25], respectively, whose definitions are given below:

$$[\mathbf{C}]_{k,l} \triangleq c(k, l) = \sqrt{\frac{2}{M}} \cos \left( \frac{\pi}{M} \left( k + \frac{1}{2} \right) \left( l + \frac{1}{2} \right) \right), \quad 0 \leq k, l \leq M-1 \quad (\text{A.6})$$

$$[\mathbf{S}]_{k,l} \triangleq s(k, l) = \sqrt{\frac{2}{M}} \sin \left( \frac{\pi}{M} \left( k + \frac{1}{2} \right) \left( l + \frac{1}{2} \right) \right), \quad 0 \leq k, l \leq M-1. \quad (\text{A.7})$$

From [25], we have the following identities (stating the unitary and Hermitian properties of  $\mathbf{C}$  and  $\mathbf{S}$ ):

$$[\mathbf{C}]^{-1} = \mathbf{C} = [\mathbf{C}]^\dagger \quad (\text{A.8})$$

$$[\mathbf{S}]^{-1} = \mathbf{S} = [\mathbf{S}]^\dagger. \quad (\text{A.9})$$

The matrices  $\mathbf{A}_0$  and  $\mathbf{A}_1$  can be expressed in terms of  $\mathbf{C}$  and  $\mathbf{S}$  as given below:

$$\mathbf{A}_0 = \sqrt{M}[\mathbf{C} + \mathbf{\Lambda}\mathbf{S}] \quad (\text{A.10})$$

$$\mathbf{A}_1 = \sqrt{M}[\mathbf{C} - \mathbf{\Lambda}\mathbf{S}] \quad (\text{A.11})$$

where  $\mathbf{\Lambda}$  is an  $M \times M$  diagonal matrix whose diagonal elements are given by  $[\mathbf{\Lambda}]_{k,k} = (-1)^k$ ,  $0 \leq k \leq M-1$ . For example, for  $M = 4$ ,  $\mathbf{\Lambda} = \text{diag}[1, -1, 1, -1]$ . Using (A.8)–(A.11), we get

$$\mathbf{A}_0^\dagger \mathbf{A}_0 = M[2\mathbf{I} + \mathbf{C}\mathbf{\Lambda}\mathbf{S} + \mathbf{S}\mathbf{\Lambda}\mathbf{C}] \quad (\text{A.12})$$

$$\mathbf{A}_1^\dagger \mathbf{A}_1 = M[2\mathbf{I} - \mathbf{C}\mathbf{\Lambda}\mathbf{S} - \mathbf{S}\mathbf{\Lambda}\mathbf{C}] \quad (\text{A.13})$$

$$\mathbf{A}_0^\dagger \mathbf{A}_1 = M[-\mathbf{C}\mathbf{\Lambda}\mathbf{S} + \mathbf{S}\mathbf{\Lambda}\mathbf{C}] = -\mathbf{A}_1^\dagger \mathbf{A}_0. \quad (\text{A.14})$$

From the definitions of  $\mathbf{C}$ ,  $\mathbf{S}$ ,  $\mathbf{\Lambda}$ , and  $\mathbf{J}$ , we can verify that  $\mathbf{\Lambda}\mathbf{S}\mathbf{J} = \mathbf{C}$ , which is a result presented in [26, eq. (56)]. This result can also be expressed as (by using the Hermitian property)

$$\mathbf{\Lambda}\mathbf{S} = \mathbf{C}\mathbf{J} \quad \text{and} \quad \mathbf{S}\mathbf{\Lambda} = \mathbf{J}\mathbf{C}. \quad (\text{A.15})$$

TABLE X  
APPENDIX A. RELATION BETWEEN THE MATRICES  
{A<sub>0</sub>, A<sub>1</sub>} AND {A<sub>0</sub>, A<sub>1</sub>}

$m$ even ( $m = 2m_1$ )	$A'_0 = (-1)^{m_1} A_1$ $A'_1 = (-1)^{(m_1-1)} A_0$
$m$ odd ( $m = 2m_1 + 1$ )	$A'_0 = (-1)^{m_1} A_0$ $A'_1 = (-1)^{m_1} A_1$

Substituting (A.15) in (A.12)–(A.14) and using the unitary property of  $C$  and  $S$  (given in equations (A.8), (A.9)), we get (A.3)–(A.5).  $\nabla \nabla \nabla$

*Part II: Proof of (17)–(19):* In this proof, we will use the results proved in part I. Table X is obtained from the definitions of {A<sub>0</sub>, A<sub>1</sub>} and {A<sub>0</sub>, A<sub>1</sub>}. Using the relations in Table X along with (A.3)–(A.5), the results in (17)–(19) can be readily verified.  $\nabla \nabla \nabla$

#### APPENDIX B

##### TWO-CHANNEL MODULATED PR FILTER BANKS

In this Appendix, we consider the cosine-modulated filter banks satisfying the PR property, for the special case  $M = 2$ , i.e., two-channel designs. We will show that these filters satisfy the same relations/properties as the filters of the two-channel PR-QMF solution given by Smith-Barnwell [6].

As mentioned in Section III, the constraint on the length of the linear-phase prototype filter is  $N = 2mM$ , where  $m > 0$ . In this case, it becomes  $N = 4m$ . From (1) and (3), we obtain the modulation equations for the analysis filters

$$h_0(n) = 2h(n) \cos \left( \frac{\pi}{4} \left( n - \frac{N-1}{2} \right) + \frac{\pi}{4} \right),$$

$$0 \leq n \leq N-1 \quad (\text{B.1})$$

$$h_1(n) = 2h(n) \cos \left( \frac{3\pi}{4} \left( n - \frac{N-1}{2} \right) - \frac{\pi}{4} \right),$$

$$0 \leq n \leq N-1. \quad (\text{B.2})$$

Using (B.1) and (B.2) it can be verified that

$$H_1(z) = -z^{-(N-1)} \tilde{H}_0(-z). \quad (\text{B.3})$$

As given in (4), the synthesis filters are related to the analysis filters by

$$F_0(z) = z^{-(N-1)} \tilde{H}_0(z) \quad (\text{B.4})$$

$$F_1(z) = z^{-(N-1)} \tilde{H}_1(z). \quad (\text{B.5})$$

Further, since the modulated PR bank prototype is designed to satisfy (20), then by Lemma 1, the polyphase component matrix  $E(z)$  is lossless. We have the following result from [8].

*Fact B.1:* Let  $E(z)$  be the polyphase component matrix of the analysis filter bank  $\{H_0(z), H_1(z), \dots, H_{M-1}(z)\}$ . If  $E(z)$  is lossless, then each of the filters  $H_k(z)$ ,  $0 \leq k \leq M-1$ , is a spectral factor of  $M$ th band filter and satisfies

$$\sum_{l=0}^{M-1} H_k(zW_M^l) \tilde{H}_k(zW_M^l) = c, \quad \forall z \quad (\text{B.6})$$

where  $c$  is a nonzero constant and  $W_M = e^{-j2\pi/M}$ .  $\diamond$

Using Fact B.1, it can be concluded that the filter  $H_0(z)$ , of a two-channel modulated PR bank, is a spectral factor of a half-band filter. Hence, it satisfies

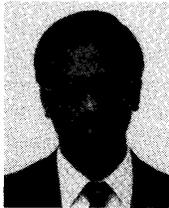
$$H_0(z) \tilde{H}_0(z) + H_0(-z) \tilde{H}_0(-z) = c, \quad \forall z. \quad (\text{B.7})$$

From (B.3)–(B.5) and (B.7), it can be verified that the filters of a two-channel modulated PR bank satisfy the same conditions as the designs in [6]. However, owing to the constraints imposed on the prototype, the two-channel cosine-modulated PR banks are only a subset of all the possible two-channel Smith-Barnwell designs (with filter length  $N = 4m$ ).

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