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MULTIRATE SYSTEMS AND FILTER BANKS

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Multirate Systems and Filter Banks

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to
my Parents
and
my wife Usha

Contents

Preface

Multirate digital signal processing techniques have been practiced by engineers for more than a decade and a half. This discipline finds applications in speech and image compression, the digital audio industry, statistical and adaptive signal processing, numerical solution of differential equations, and in many other fields. It also fits naturally with certain special classes of time-frequency representations such as the short-time Fourier transform and the wavelet transform, which are useful in analyzing the time-varying nature of signal spectra.

Over the last decade, there has been a tremendous growth of activity in the area of multirate signal processing, perhaps triggered by the first book in this field [Crochiere and Rabiner, 1983]. Particularly impressive is the amount of new literature in digital filter banks, multidimensional multirate systems, and wavelet representations. The theoretical work in multirate filter banks appears to have reached a level of maturity which justifies a thorough, unified, and in-depth treatment of these topics. This book is intended to serve that purpose, and it presents the above mentioned topics under one cover. Research in the areas of multidimensional systems and wavelet transforms is still proceeding at a rapid rate. We have dedicated one chapter to each of these, in order to bring the reader up to a point where research can be begun.

I have always believed that it is important to appreciate the generality of principles and to obtain a solid theoretical foundation, and my presentation here reflects this philosophy. Several applications are discussed throughout the book, but the general principles are presented without bias towards specific application-oriented detail.

The writing style here is very much in the form of a *text.* Whenever possible I have included examples to demonstrate new principles. Many design examples and complete design rules for filter banks have been included. Each chapter includes a fairly extensive set of homework problems (totaling over 300). The solutions to these are available to instructors, from the publisher. Tables and summaries are inserted at many places to enable the reader to locate important results conveniently. I have also tried to simplify the reader's task by assigning separate chapters for more advanced material. For example, Chap. 11 is dedicated to wavelet transforms, and Chap. 14 contains detailed developments of many results on paraunitary systems. Whenever a result from an advanced chapter (for example, Chap. 14) is used in an earlier chapter, this result is first stated clearly within the context of use, and the reader is referred to the appropriate chapter for proof.

The text is self-contained for readers who have some prior exposure to digital signal processing. A one-term course which deals with sampling, discrete-time Fourier transforms, z-transforms, and digital filtering, is sufficient. In Chap. 2 and

3 a brief review of this material is provided. A thorough exposition can be found in a number of references, for example, [Oppenheim and Schafer, 1989]. Chapter 3 also contains some new material, for example, *eigenfilters*, and detailed discussions on allpass filters, which are very useful in multirate system design.

A detailed description of the text can be found in Chap. 1. Chapters 2 and 3 provide a brief review of signals, systems, and digital filtering. Chapter 4, which is the first one on multirate systems, covers the fundamentals of multirate building blocks and filter banks, and describes many applications. Chapter 5 introduces multirate filter banks, laying the theoretical foundation for alias cancelation, and elimination of other errors. The first two sections in Chap. 4 and 5 contain material overlapping with [Crochiere and Rabiner, 1983]. Most of the remaining material in these chapters, and in the majority of the chapters that follow, have not appeared in this form in text books.

Chapters 6 to 8 provide a deeper study of multirate filter banks, and present several design techniques, including those based on the so-called paraunitary matrices. (These matrices play a role in the design of many multirate systems, and are treated in full depth in Chap. 14.) Chapters 9 to 12 cover special topics in multirate signal processing. These include roundoff noise effects (Chap. 9), block filtering, periodically time varying systems and sampling theorems (Chap. 10), wavelet transforms (Chap. 11) and *multidimensional* multirate systems (Chap. 12). Chapters 13 and 14 give an in-depth coverage of multivariable linear systems and lossless (or paraunitary) systems, which are required for a deeper understanding of multirate filter banks and wavelet transforms.

There are five appendices which serve as references as well as supplementary reading. Three of these are review-material (matrix theory, random processes, and Mason's gain formula). Two of the appendices contain results directly related to filter bank systems. One of these is a technique for spectral factorization; the other one analyzes the effects of quantization of subband signals.

Many of these chapters have been taught at Caltech over the last three years. This text can be used for teaching a one, two, or three term (quarter or semester) course on one of many possible topics, for example, multirate fundamentals, multirate filter banks, wavelet representation, and so on. There are many homework problems. The instructor has a great deal of flexibility in choosing the topics, but I prefer not to bias him or her by providing specific course outlines here.

In summary, I have endeavored to produce a text which is useful for the classroom, as well as for self-study. It is also hoped that it will bring the reader to a point where he/she can start pursuing research in a vast range of multirate areas. Finally, I believe that the text can be comfortably used by the practicing engineer because of the inclusion of several design procedures, examples, tables, and summaries.

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